Recent Progress in Few- to Many-Nucleon Physics

Outline

- **Hamiltonian: Two- and Three- Nucleon Interactions**
- Zero Q: **Spectra, Static Properties, Larger Systems**
- Very Low Q: **Low Energy Reactions: EW capture, PV, ...**
- Low Q: Eletron Scattering: F.F., (e,e'p), (e,e')

Collaborators

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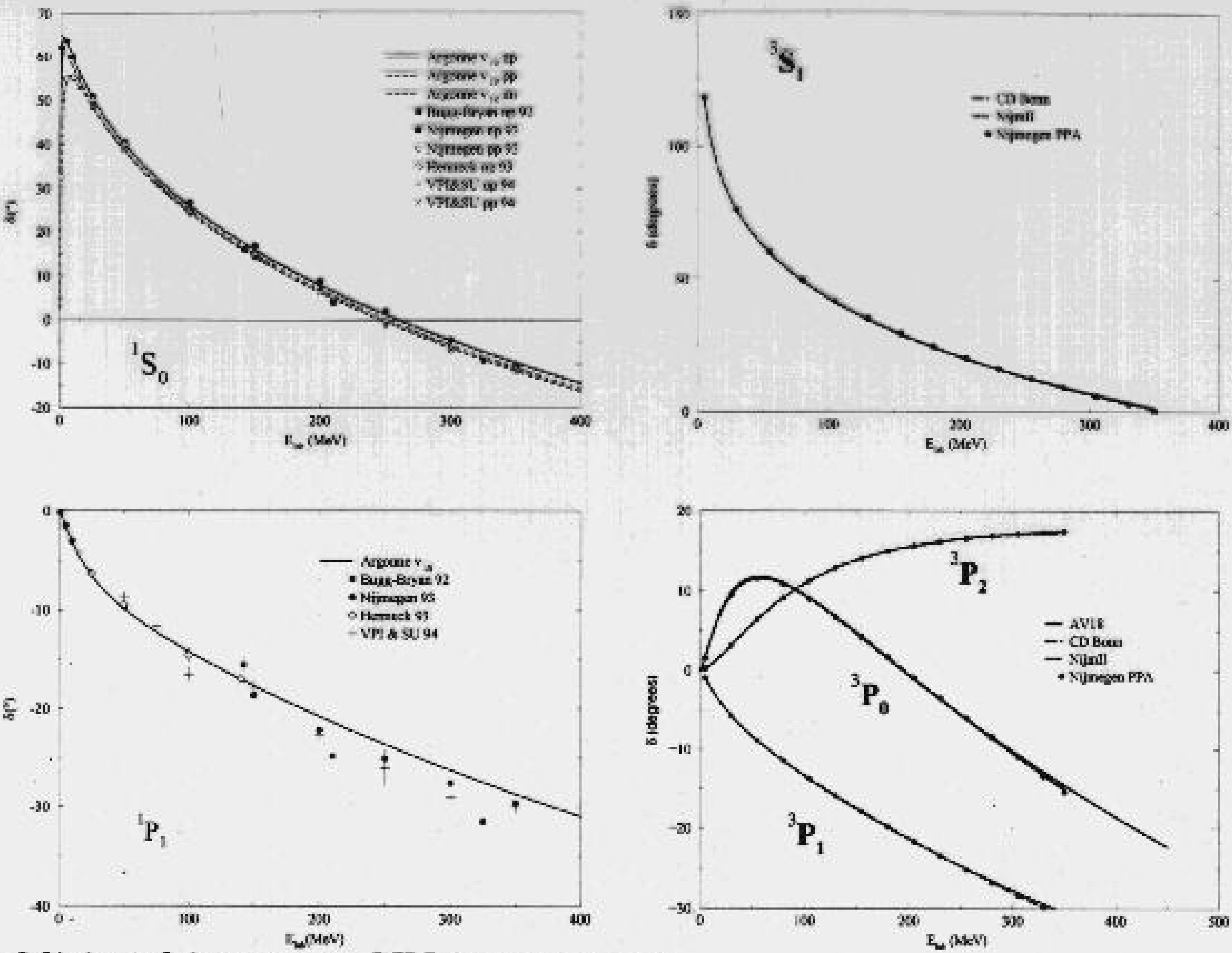
Interaction: AV18 NN + TNI

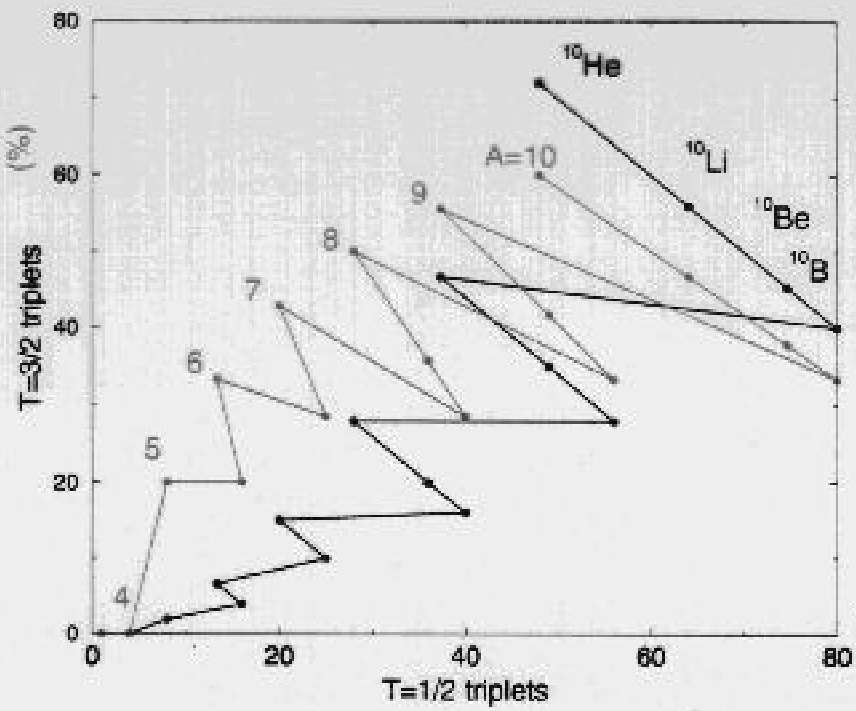


Additional inputs to NN interaction:

- Fits to more recent IUCF data
- Chiral 2-pion exchange interaction
- Want a (maximally) local interaction

Additional Information in A>4 Nuclei:

- Sensitivity to P-wave interactions
- Bound-state or quasi-bound Excitations
- L·S splittings
- Neutron-Rich systems

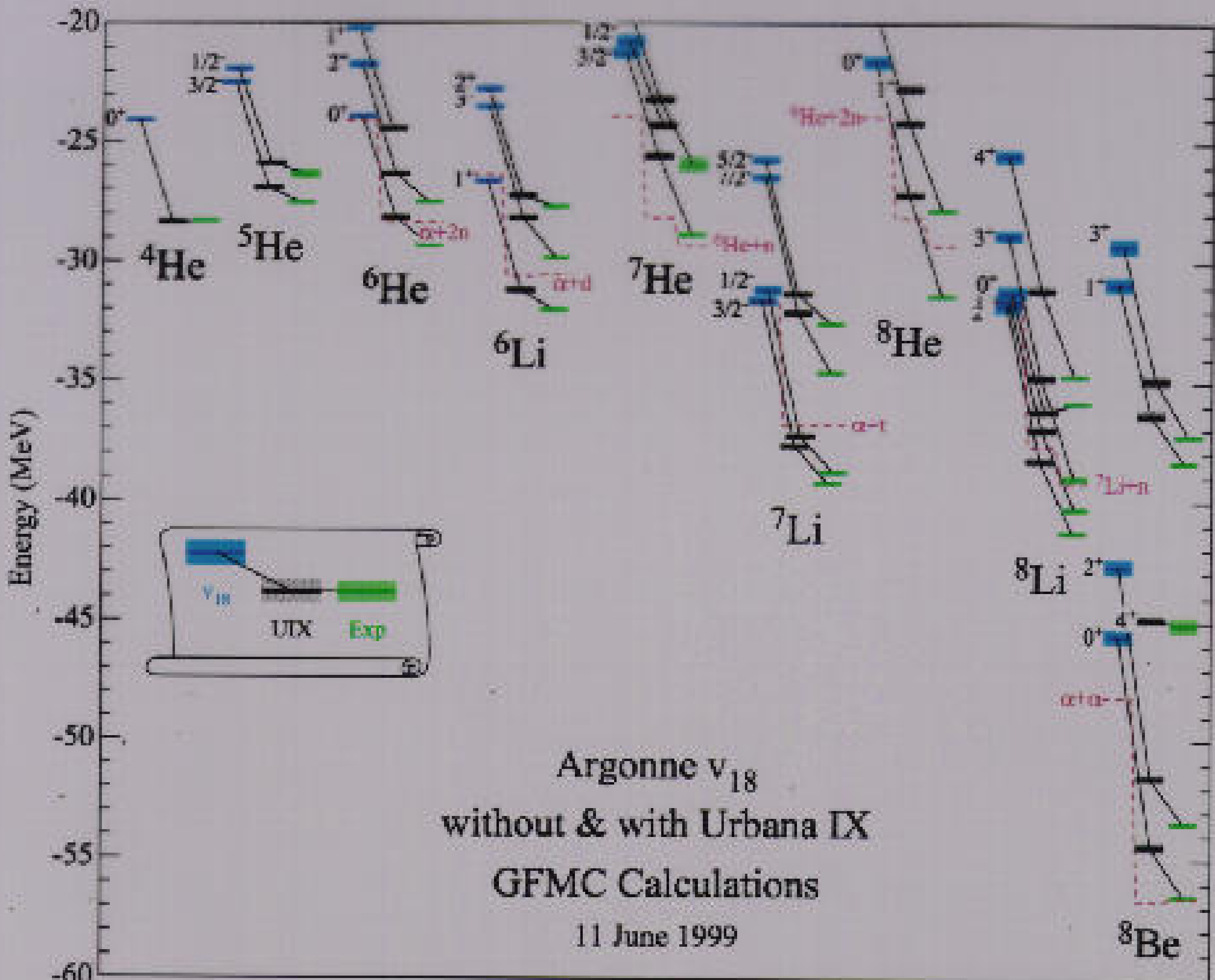


Host of interesting problems:

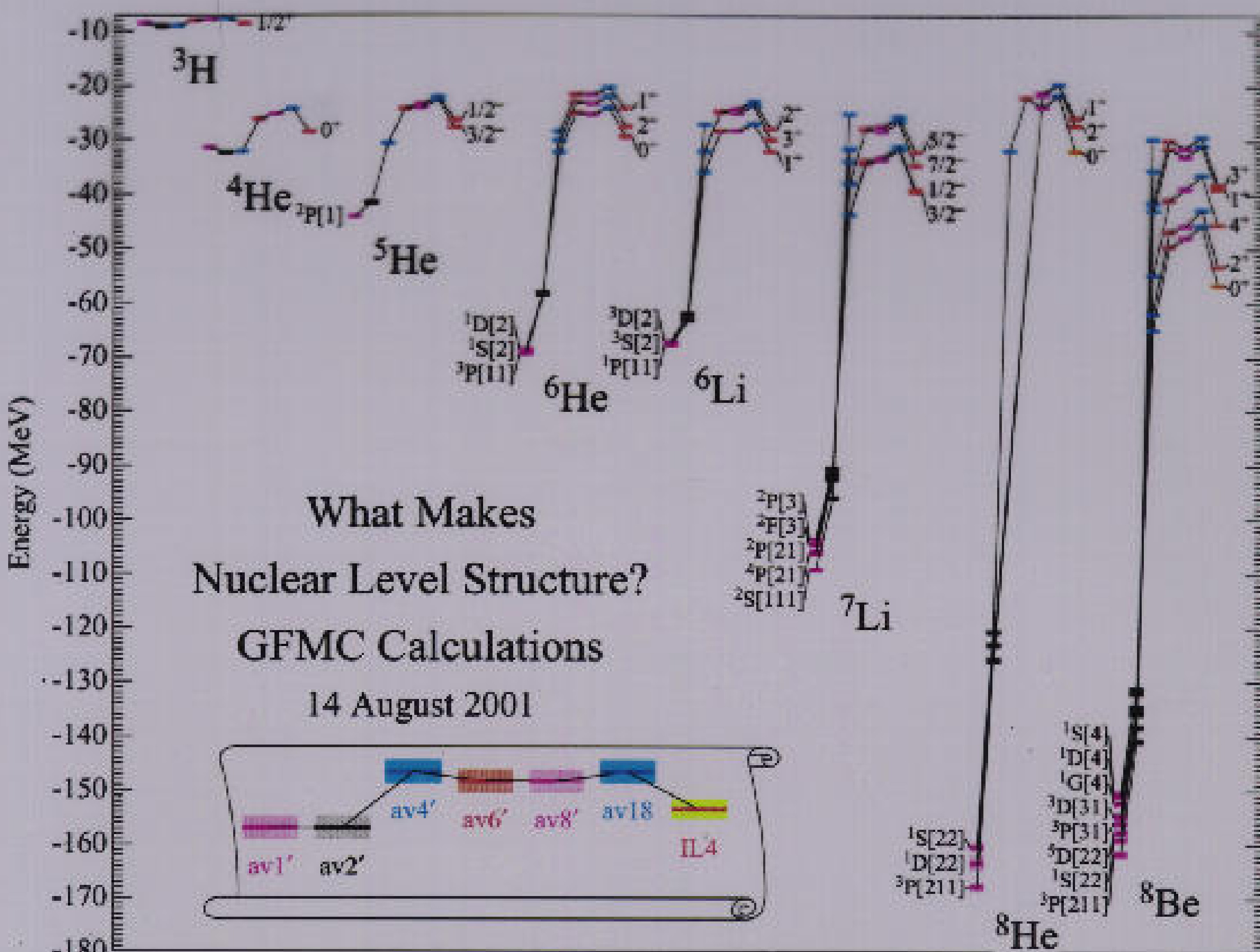
- Beta Decay Studies - Standard model tests
- Parity Violation
- Low Energy Astrophysical Reactions

All require accurate Hamiltonian, ...

Light Nuclear Spectra - AV18 + UIX

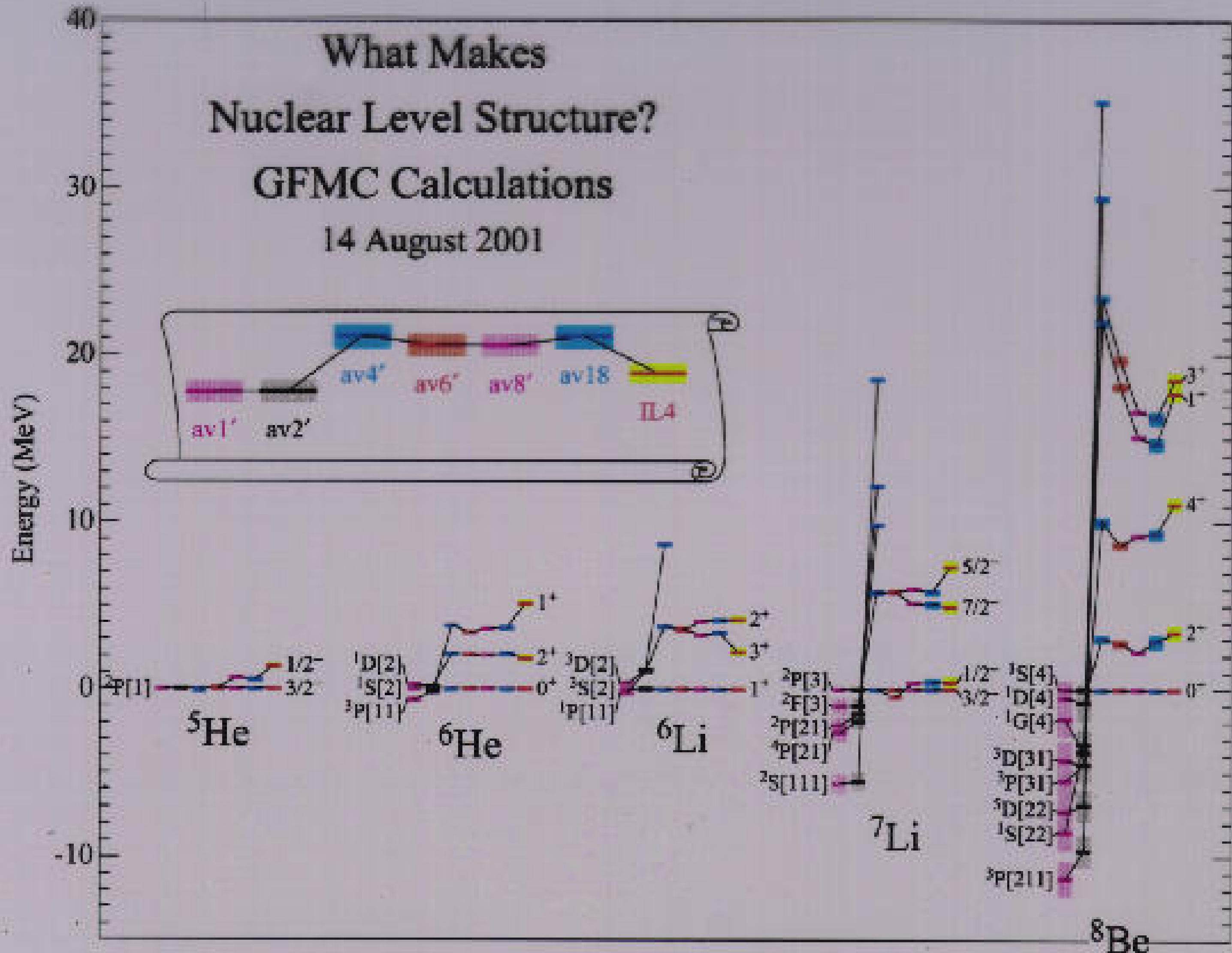


Nuclear Spectra Inputs(Pieper and Wiringa)



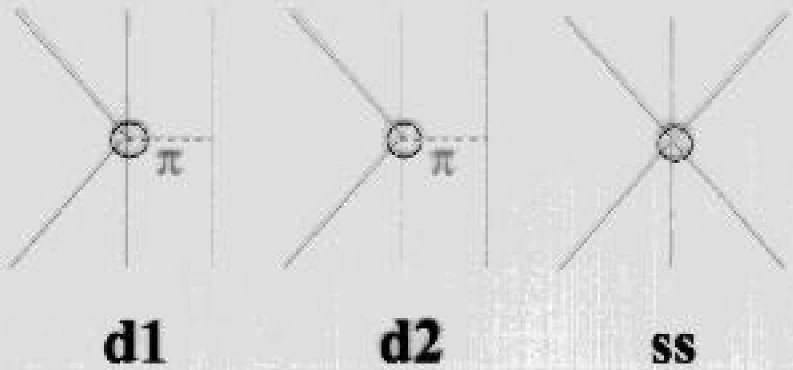
av1′, av2′ *results are preliminary*

Nuclear Spectra Inputs(Pieper and Wiringa)



av1′, av2′ results are preliminary

TNI - Texas Los Alamos - χPT

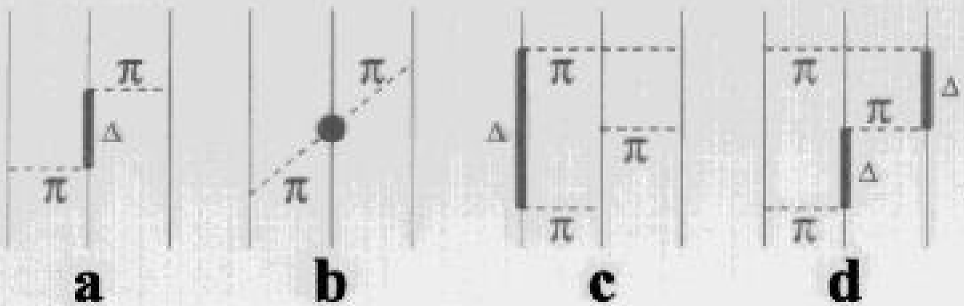


$$
\begin{aligned}
V_{ijk} &= \frac{d_1}{(2\pi)^6}\frac{g_A}{2f_\pi^2}\sigma_1\cdot\mathbf{Q}'\sigma_2\cdot\mathbf{Q}'\frac{1}{Q'^2+m_\pi^2}\tau_1\cdot\tau_2+... \\
&+ \frac{d_2}{(2\pi)^6}\frac{g_A}{4f_\pi^2}\sigma_1\times\sigma_3\cdot\mathbf{Q}'\sigma_2\cdot\mathbf{Q}'\frac{1}{Q'^2+m_\pi^2}\tau_1\cdot\tau_2\times\tau_3+... \\
&+ \text{short} - \text{short range terms}
\end{aligned}
$$

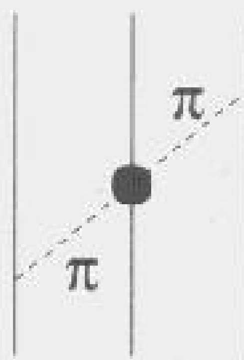
- short-short terms would affect dense snm
- d1 term that 'enhances' OPEP increases A_y.
- Fix coefficients by three-nucleon scattering? Note different behavior at 3 vs. 50 MeV; tensor observables,
- Only d_1 terms and 2π TNI b term important in neutron-rich systems
- Can Nd scattering, light nuclear spectra and dense matter be described simultaneously? Requires cancellation in light nuclei and symmetric matter between short-short interactions and d_1 term.

Three-Nucleon Interactions - Illinois Models

$$V_{ijk} = A_{2\pi}^{PW} O_{ijk}^{2\pi,PW} + A_{2\pi}^{SW} O_{ijk}^{2\pi,SW} + A_{3\pi}^{\Delta R} O_{ijk}^{3\pi,\Delta R} + A_R O_{ijk}^{R}$$



- Strengths of the four terms are varied to fit the observed energies of ~ 16 states of up to 8 nucleons.
- Different models - w/ and w/o s-wave πN and different couplings in (a)

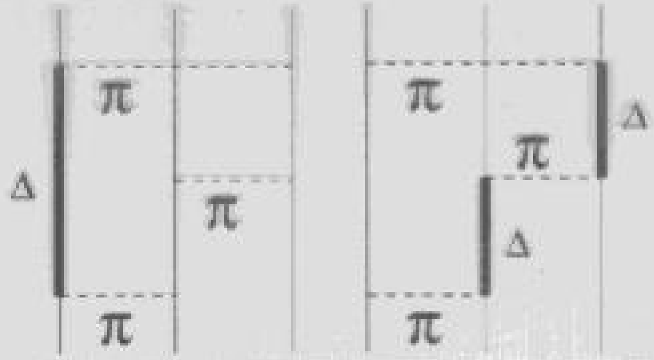


$V^{2\pi,SW}$ in the TM model

$$B(\mathbf{r}_{ij},\mathbf{r}_{jk})\ \{\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j,\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k\}\left\{(S_{ij}+\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j),(S_{jk}+\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_k)\right\}\ ,$$

Short-range terms containing Z_0' omitted

$V^{3\pi,\Delta R}$



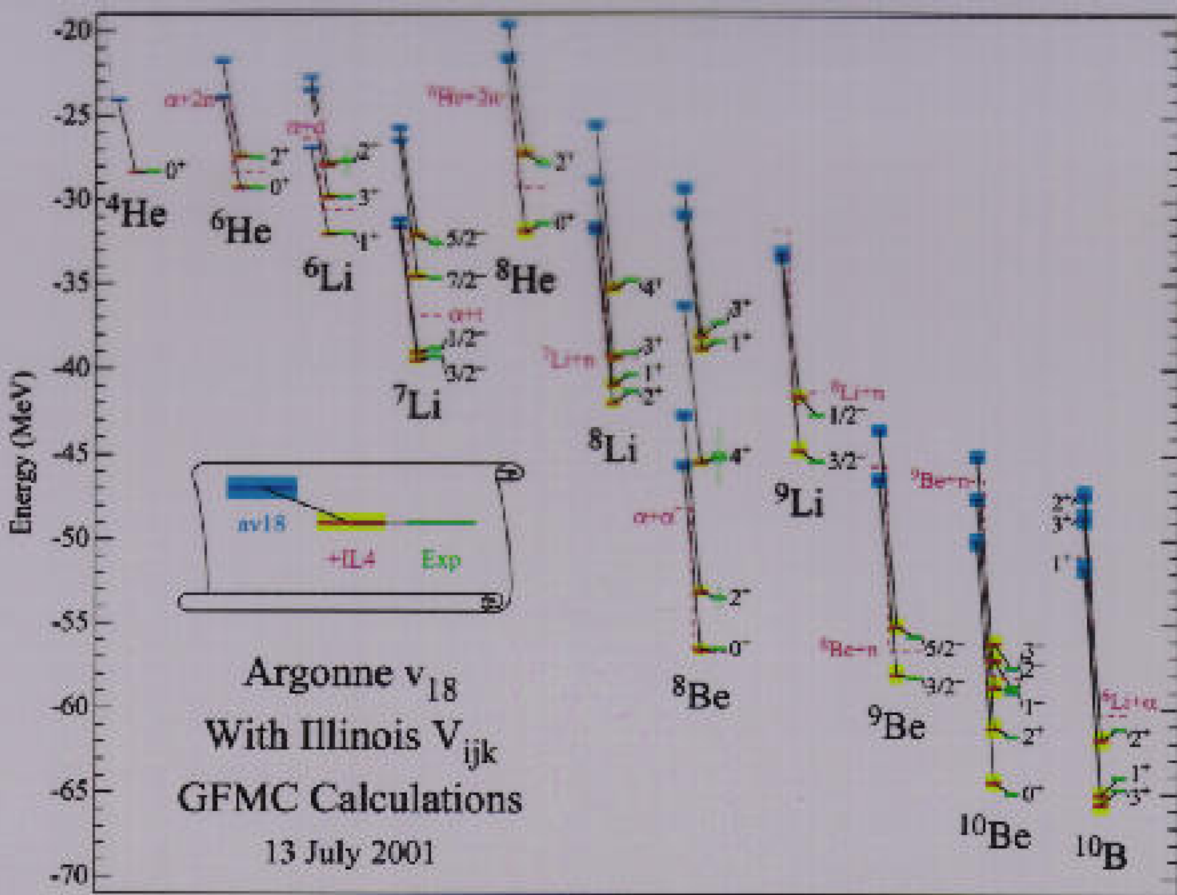
$$V_{1,ijk}^{3\pi,\Delta R} = \sum_{cyc} \frac{1}{(m_\Delta - m_N)^2} \times$$
$$(v^{\pi}_{\Delta N\to NN}(ik)\; v^{\pi}(jk)\; v^{\pi}_{NN\to\Delta N}(ij) \;+\; j \rightleftharpoons k)$$

$$V_{2,ijk}^{3\pi,\Delta R} = \sum_{cyc} \frac{1}{(m_\Delta - m_N)^2}$$
$$+ \quad (v^{\pi}_{N\Delta\to NN}(ik)\; v^{\pi}_{\Delta N\to N\Delta}(jk)\; v^{\pi}_{NN\to N\Delta}(ij) \;+\; j \rightleftharpoons k) \;,$$

$$O_{ijk}^{3\pi,\Delta R} \approx \frac{50}{3} S^I_\tau S^I_\sigma + \frac{26}{3} A^I_\tau A^I_\sigma$$

(Comparatively) Large Effect in Isospin 3/2

Interaction:

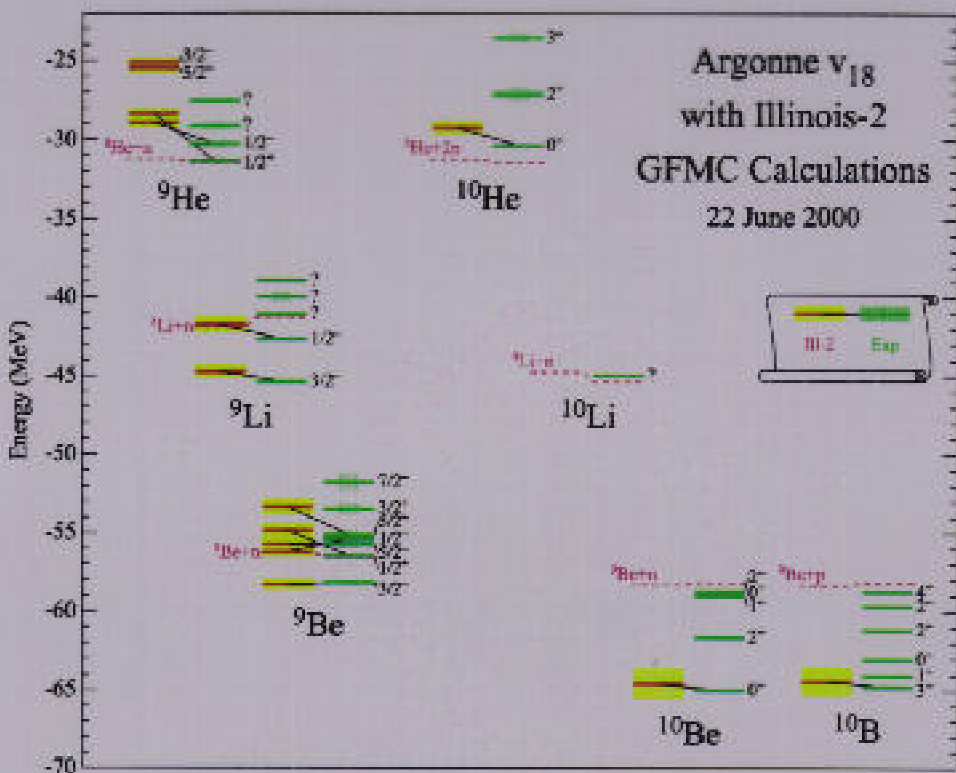


^{10}Be unnatural parity state calculations are preliminary

Caveats:

- Chiral Two-Pion Exchange in NN interaction Models
- Alternative Three-Nucleon Interactions
- To what extent do these affect physical observables?

Interaction:
Mass 9-10 Spectra
from Illinois Vijk

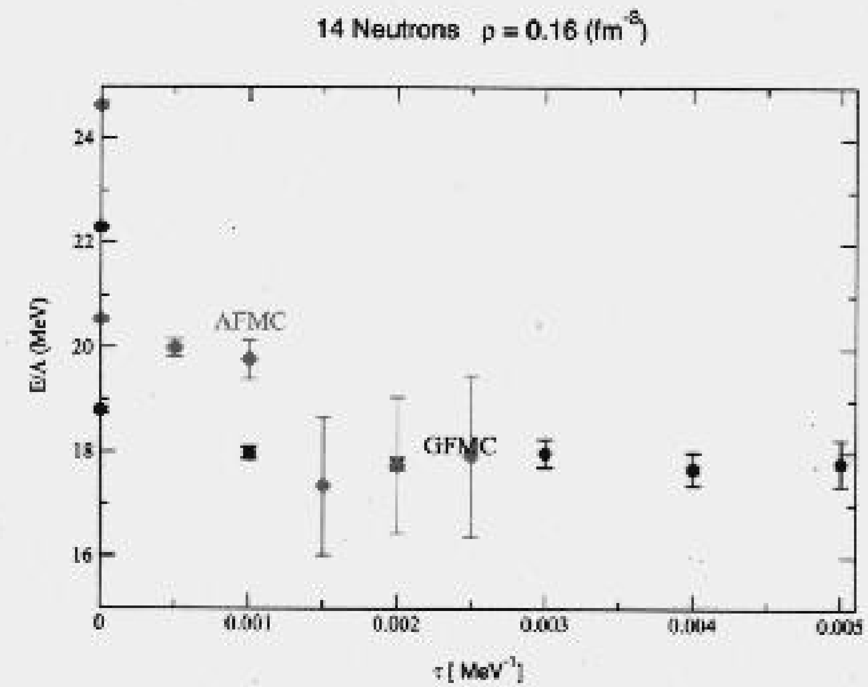


What About Larger Systems?

- Auxiliary-Field Diffusion Monte Carlo (Schmidt and Fantoni)
- Samples Spins and Isospins as well as coordinates
- Relies on Constrained Path Approximation (positive overlap with trial fn)

Test Problem: 14 Neutrons

- Test Neutron Matter Equation of State
- Importance of new TNI
- Spin Susceptibility, ...



Very Low Q: Capture Reactions, PV, ...

Below breakup methods almost identical to bound state

- **VMC - implement correct asymptotic boundary conditions; can use variational principle**
- **GFMC** - boundary conditions on wvfn implemented with image methods.

Solar neutrino production:

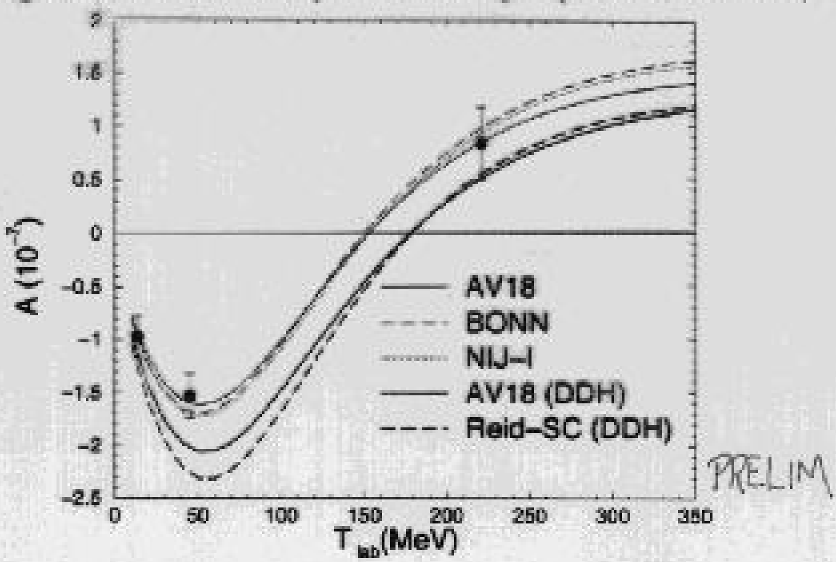
- pp capture ~0.2%
  Very accurate limits on pp capture rate arise from incorporating constraints on tritium β-decay. Now done with both 'traditional' and chiral effective field theory (Kubodera, Schiavilla) models of the weak currents.
- hep capture
  Importance of p-wave capture; also done with both sets of currents - $\approx$ 30% uncertainty.

Big-Bang Nucleosynthesis (K. Nollett's talk):

- α-d capture
- Capture in A=7

Hadronic Parity Violation

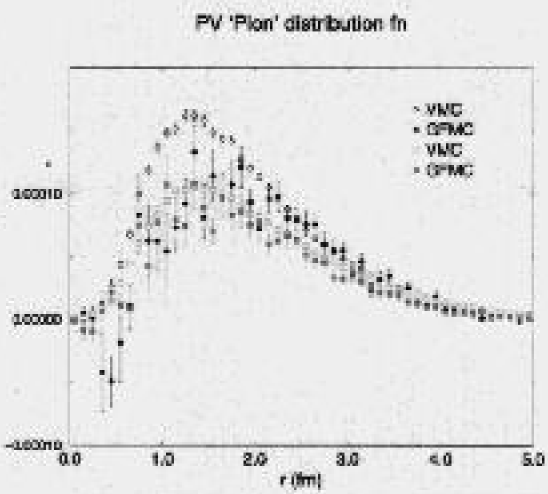
PP Longitudinal Asymmetry (TRIUMF, ...)

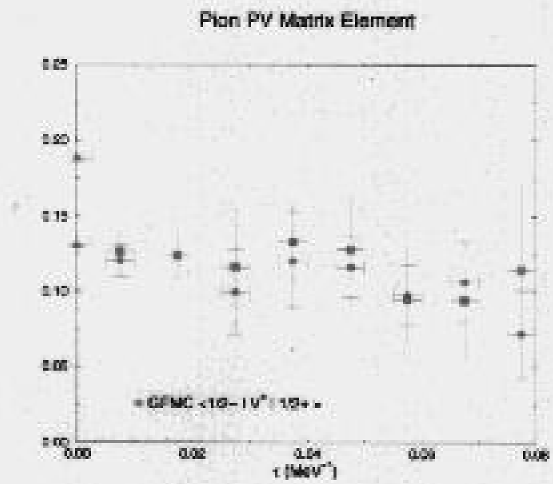


Parity-Violation in $\vec{n}-\alpha$

Radial Dependence

Matrix Element Convergence





Similar efforts underway in np $\rightarrow$ d gamma; $\vec{p}-\alpha$; etc.

Electroweak Transitions:
Low-Modest Q

Current Operators

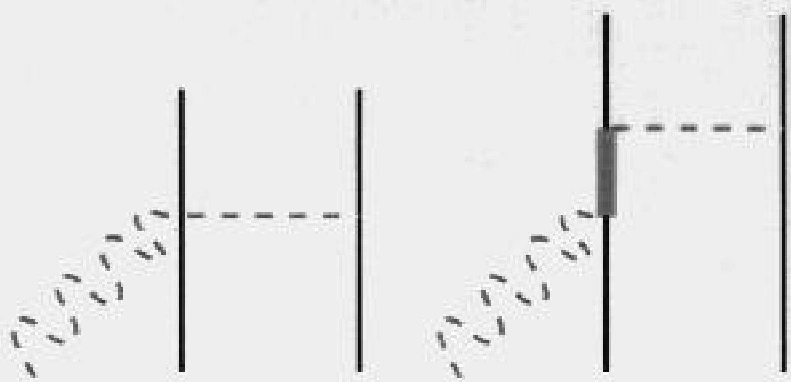
$$
\begin{aligned}
\rho(\mathbf{q}) &= \sum_i \rho_i^1 + \sum_{i<j>} \rho_{ij}^2 + \ldots \\
\rho_i^{(1)}(\mathbf{q}) &= \rho_{i,\mathrm{NR}}^{(1)}(\mathbf{q}) + \rho_{i,\mathrm{RC}}^{(1)}(\mathbf{q}) \;, \\
\rho_{i,\mathrm{NR}}^{(1)}(\mathbf{q}) &= \epsilon_i \, \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_i} \\
\rho_{i,\mathrm{RC}}^{(1)}(\mathbf{q}) &= \left(\frac{1}{\sqrt{1+Q^2/4m^2}} - 1 \right) \epsilon_i \, \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_i} \\
&\quad - \frac{\mathrm{i}}{4m^2} \left(2\,\mu_i - \epsilon_i\right) \mathbf{q}\cdot(\sigma_i \times \mathbf{p}_i)\, \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_i}
\end{aligned}
$$

$$
\begin{aligned}
\rho_{ij,\pi}(\mathbf{k}_i,\mathbf{k}_j) &= \frac{3}{2m}\Big[\left[F_1^S(Q^2)\tau_i\cdot\tau_j + F_1^V(Q^2)\tau_{z,j} \right] \\
&\quad v_\pi(k_j)\sigma_i\cdot\mathbf{q}\,\sigma_j\cdot\mathbf{k}_j \\
&\quad + \left[F_1^S(Q^2)\tau_i\cdot\tau_j + F_1^V(Q^2)\tau_{z,i} \right] \\
&\quad v_\pi(k_i)\sigma_i\cdot\mathbf{k}_i\,\sigma_j\cdot\mathbf{q} \Big] \;,
\end{aligned}
$$

EM Currents

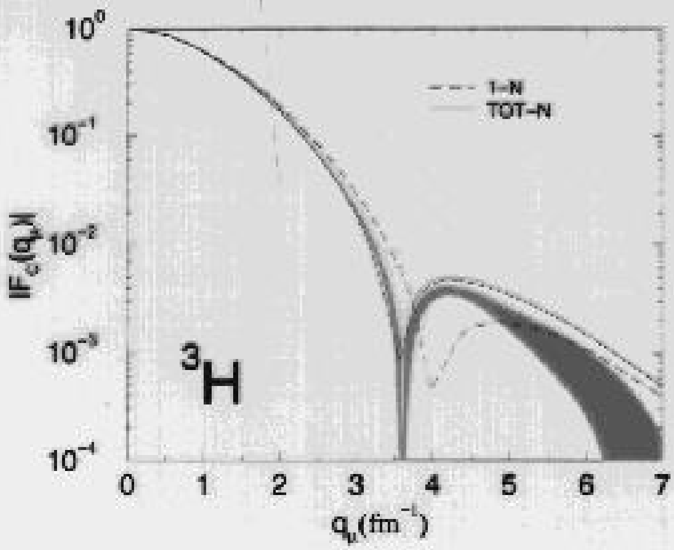
$$\mathbf{j}_i^{(1)}(\mathbf{q}) = \frac{1}{2m}\epsilon_i \left\{\mathbf{p}_i \,,\, \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_i}\right\} - \frac{\mathrm{i}}{2m}\mu_i \, \mathbf{q} \times \sigma_i \, \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}_i}$$

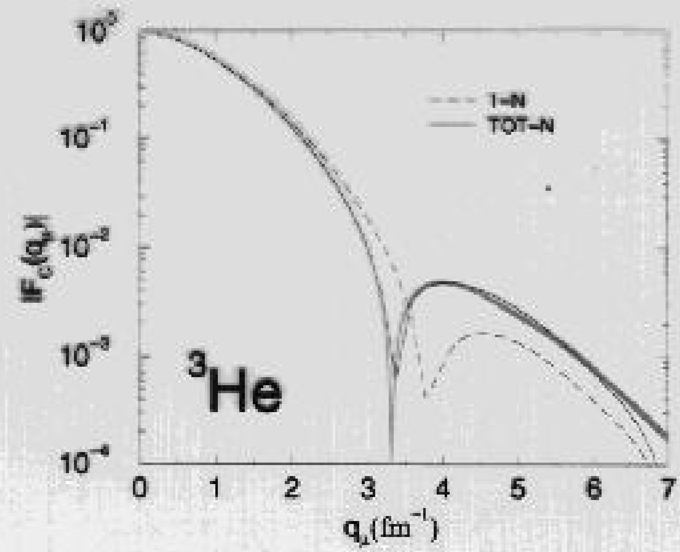
$$\mathbf{j}_{ij,\pi}^{(2)}(\mathbf{k}_i, \mathbf{k}_j) = 3\mathrm{i}(\tau_i \times \tau_j)_z \, G_E^V(Q^2) \left[v_\pi(k_j)\sigma_i(\sigma_j \cdot \mathbf{k}_j) - v_\pi(k_i)\sigma_j(\sigma_i \cdot \mathbf{k}_i)\right]$$



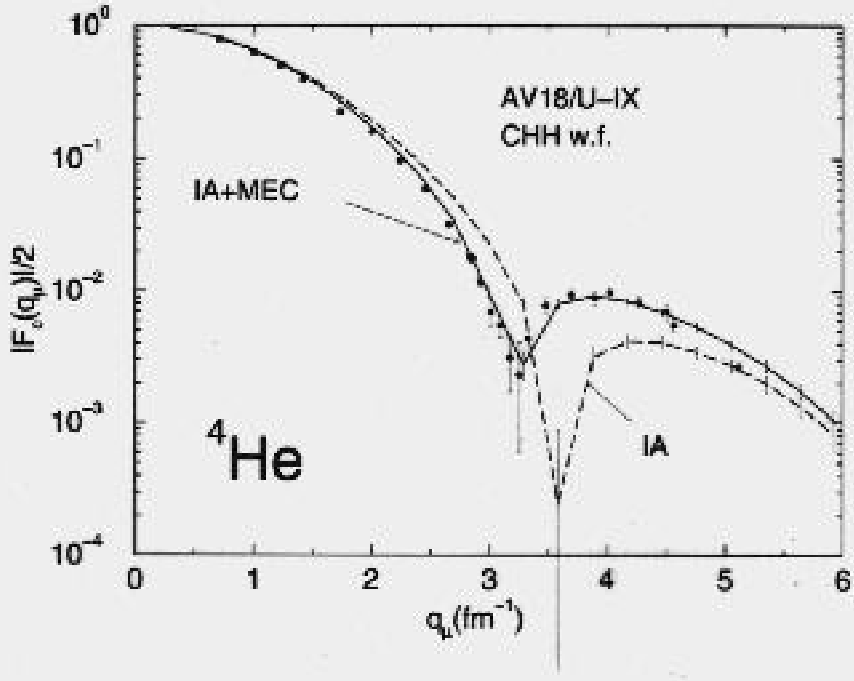
$$\mathbf{j}_{\Delta_{\mathrm{PT}},ij} = \mathbf{j}_i(\mathbf{q}; \Delta \to N)\frac{v_{NN\to\Delta N,ij}}{m - m_\Delta} + \frac{v_{\Delta N\to NN,ij}}{m - m_\Delta}\mathbf{j}_i(\mathbf{q}; N \to \Delta) + i \rightleftharpoons j$$

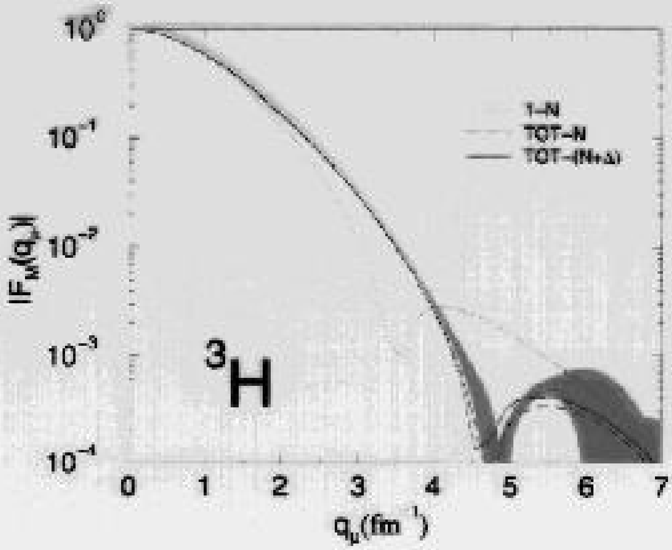
...



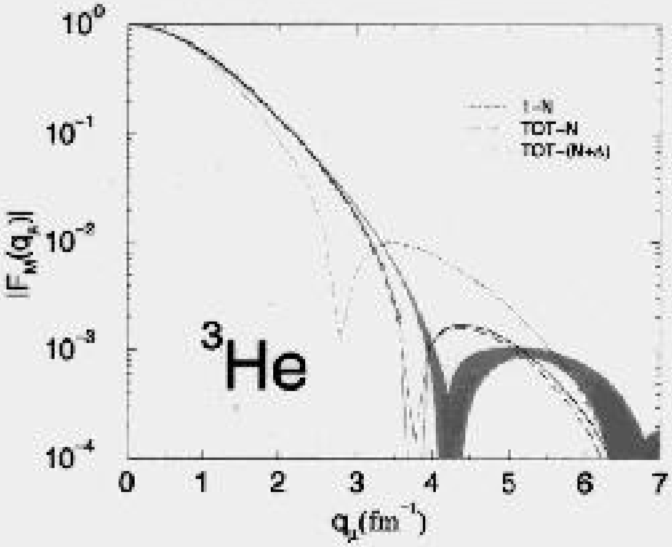


CHARGE FORM FACTOR





1-N
TOT-N
TOT-(N+Δ)
$|F_M(q_\mu)|$
^{3}H
q_μ(fm^{-1})



1-N
TOT-N
TOT-(N+Δ)
$|F_M(q_\mu)|$
^{3}He
q_μ(fm^{-1})

A=6 Form Factors

Fig.2 Wiringa & Schiavilla

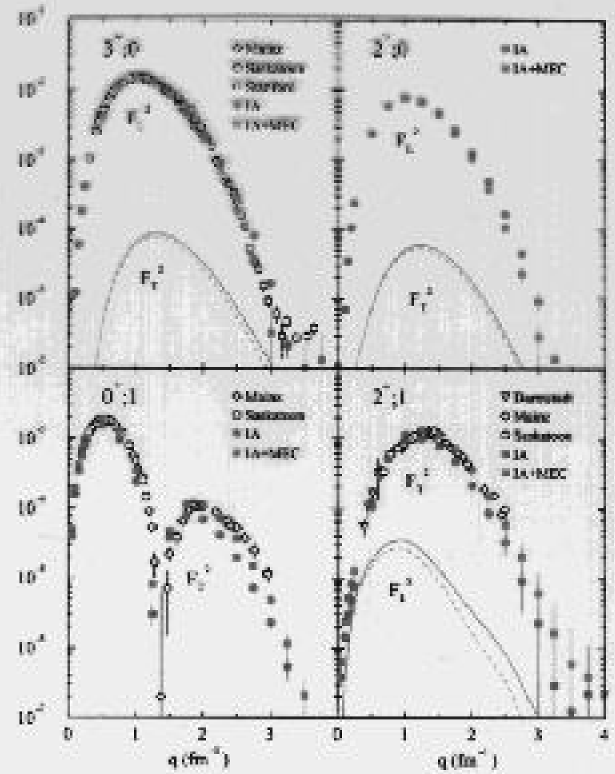
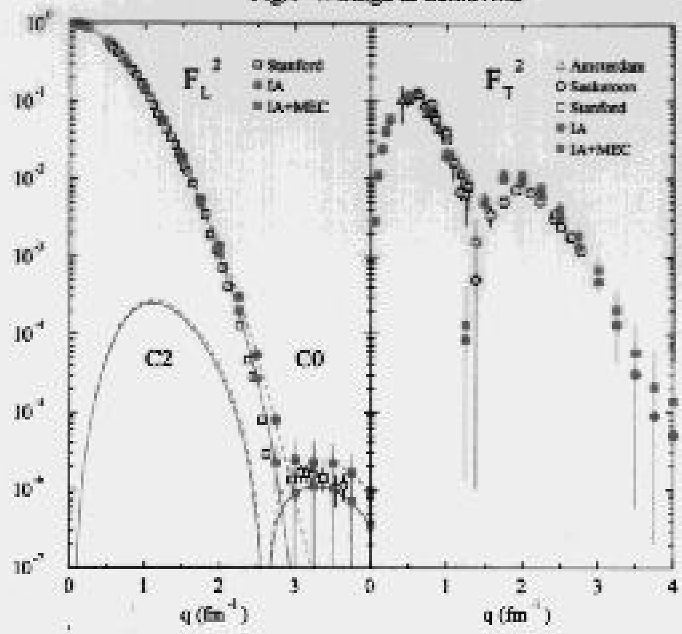


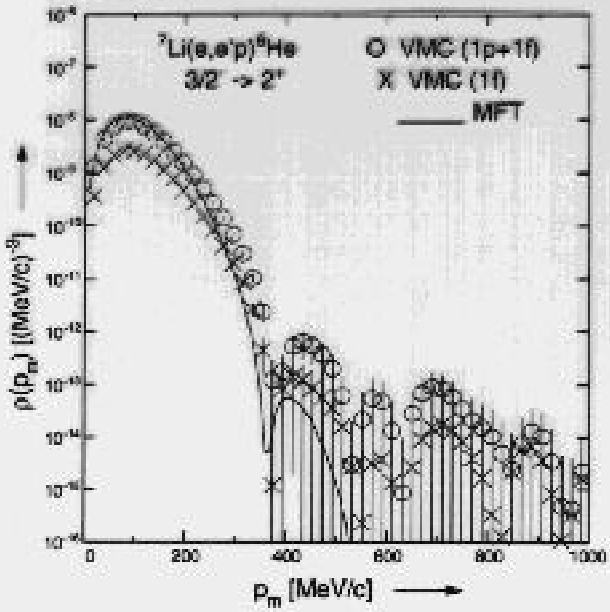
Fig.1 Wiringa & Schiavilla

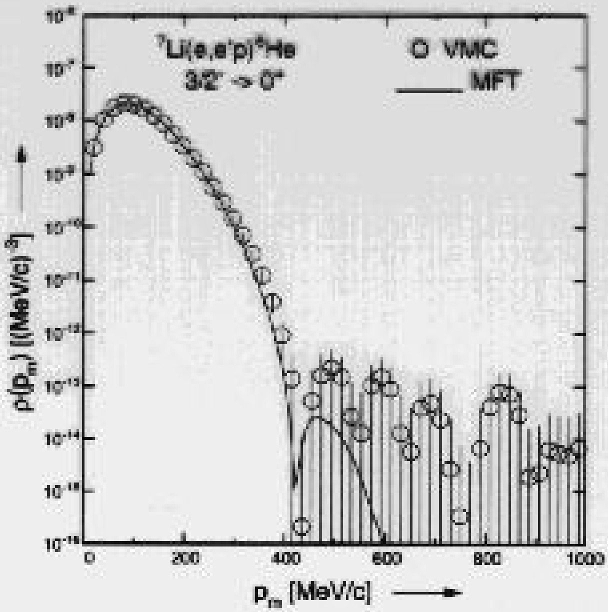


Caveats

- Relativistic Treatment consistent with charge operator
- Experimental constraints on currents beyond Pion
- Electroweak Currents...

Electron Scattering: (e,e'p)





Calculation:

- Microscopic calculations of nuclear wvfns, overlaps
- Optical Potential Treatment
- Challenges: Microscopic determination of exclusive scattering, studies of energy dependence, etc.

Inclusive Response

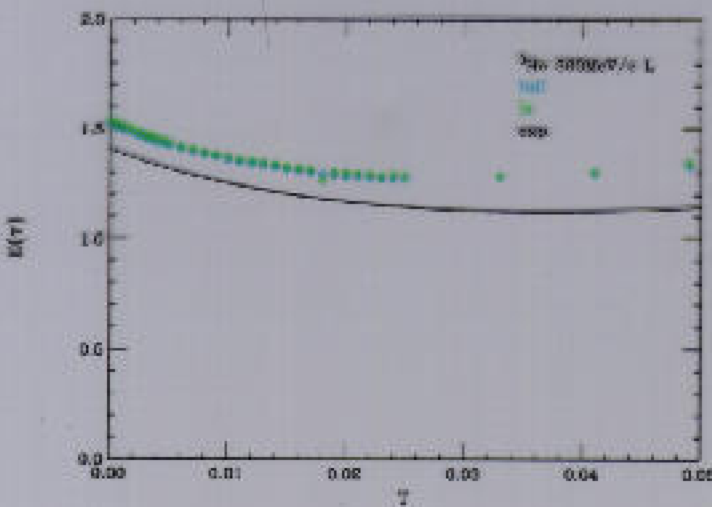
Longitudinal / Transverse Sum Rules

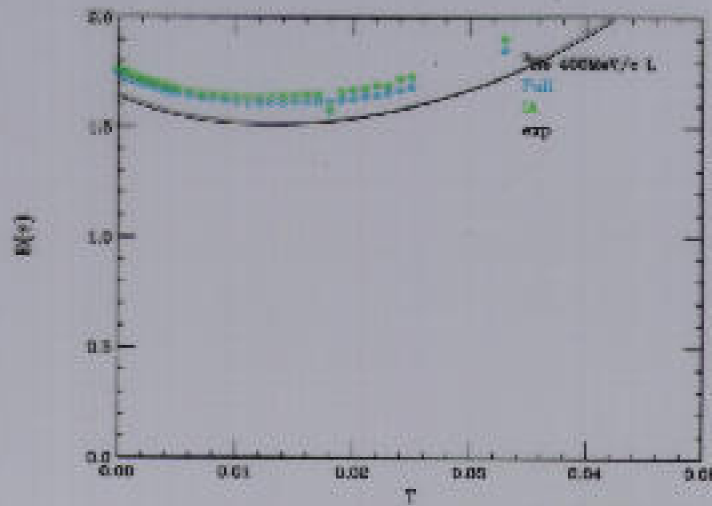
$$
\begin{aligned}
S_L(q) &= \langle 0|\rho^\dagger(\mathbf{q})\rho(\mathbf{q})|0> - |\langle 0_\mathbf{q}|\rho(\mathbf{q})|0\rangle|^2 \\
S_T(q) &= \langle 0|\mathbf{J}^\dagger(\mathbf{q})\mathbf{J}(\mathbf{q})|0> - |\langle 0_\mathbf{q}|\mathbf{J}(\mathbf{q})|0\rangle|^2
\end{aligned}
$$

Euclidean Response

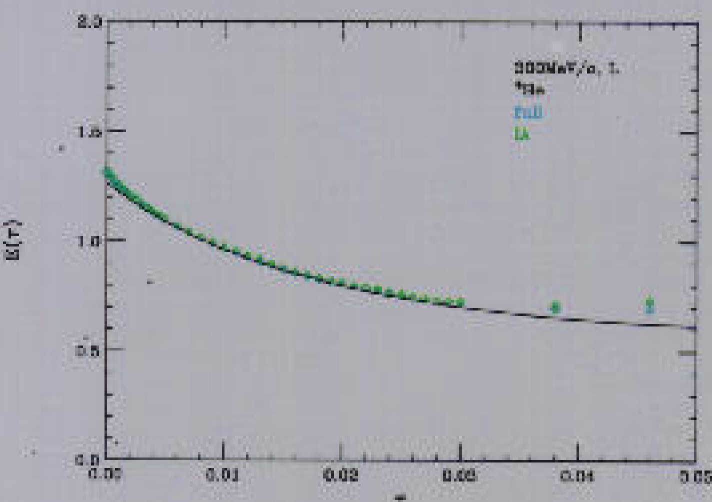
$$
\begin{aligned}
E_L(q) &= \langle 0|\rho^\dagger(\mathbf{q})\exp[-(H - E_0 - q^2/(2m))\tau]\rho(\mathbf{q})|0> \\
&\quad -|\langle 0_\mathbf{q}|\rho(\mathbf{q})|0\rangle|^2 \exp[-E_r\tau] \\
&= \int d\omega\ S_L(q,\omega)\ \exp(-\omega\tau) \\
E_T(q) &= \langle 0|\mathbf{J}^\dagger(\mathbf{q})\exp[-(H - E_0 - q^2/(2m))\tau]\mathbf{J}(\mathbf{q})|0> \\
&\quad -|\langle 0_\mathbf{q}|\mathbf{J}(\mathbf{q})|0\rangle|^2 \exp[-(E_r - q^2/(2m)\tau] \\
&= \int d\omega\ S_T(q,\omega)\ \exp(-(\omega - q^2/(2m)\tau)
\end{aligned}
$$

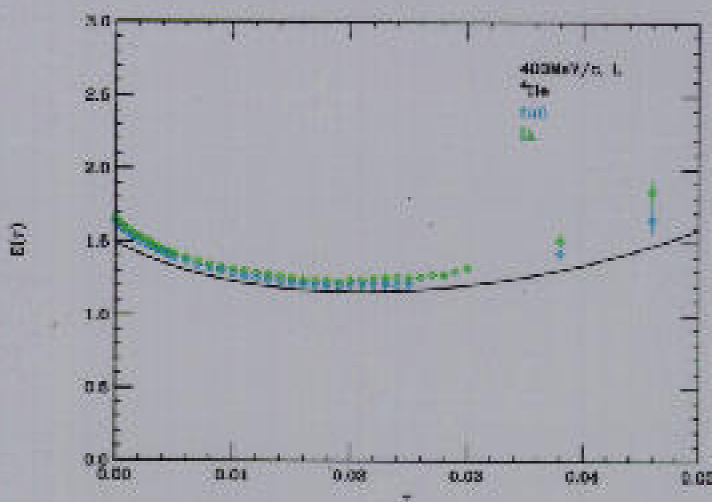
^{3}He Longitudinal Theory vs. Exp



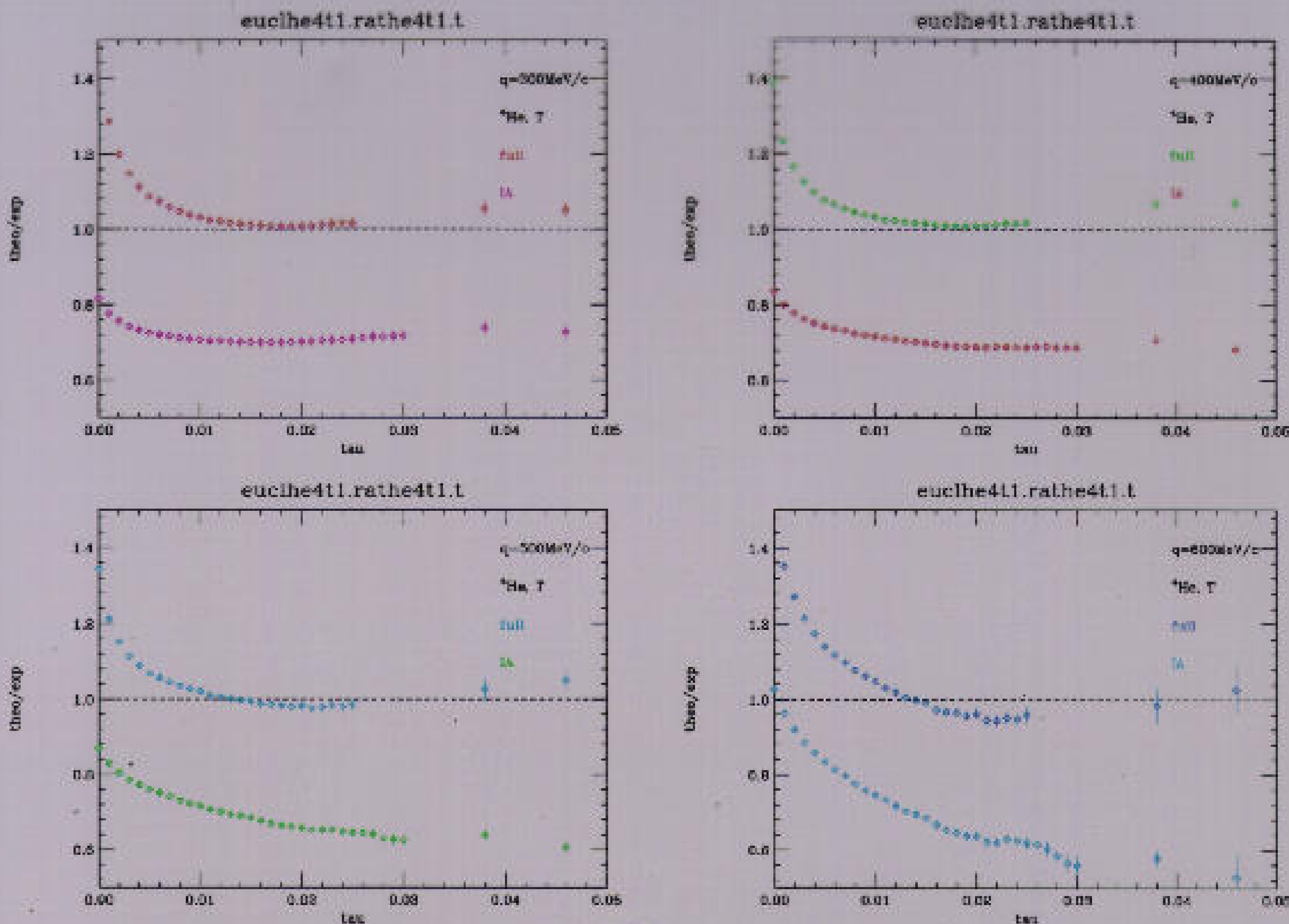


^{4}He Longitudinal Theory vs. Exp

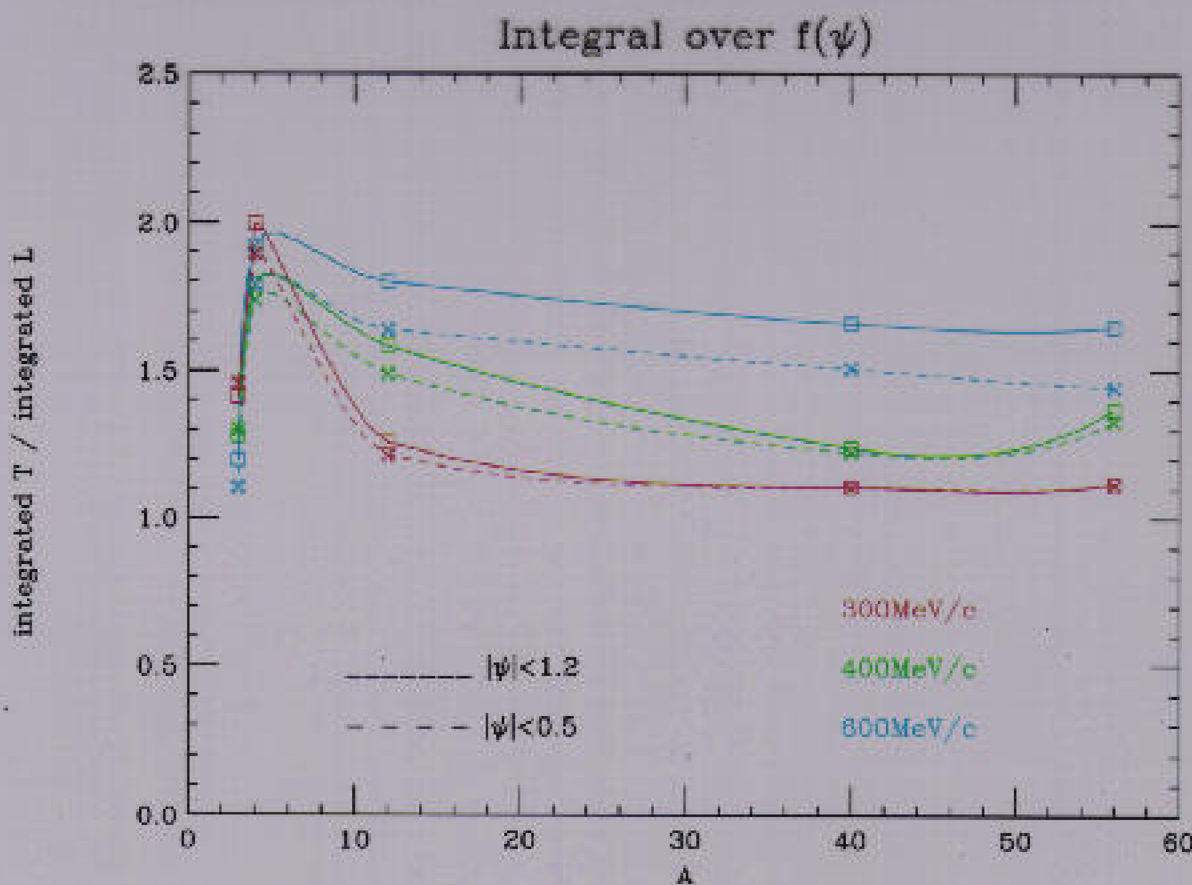




^{4}He Transverse Theory / Exp

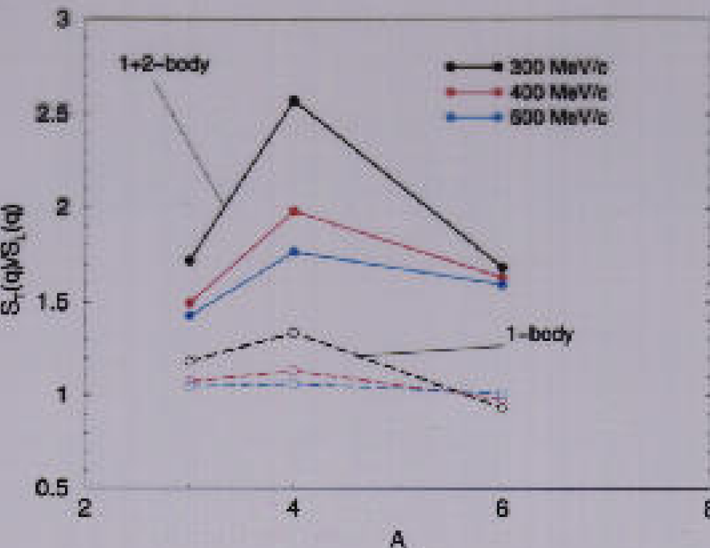


Inclusive Response: Integrated Transverse / Longitudinal Strength



QE scattering

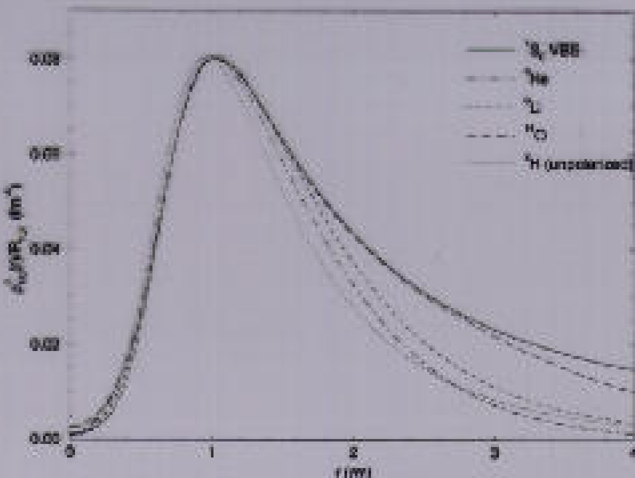
Inclusive Response Beyond A=4
'Excess' Strength from T/L Sum Rules



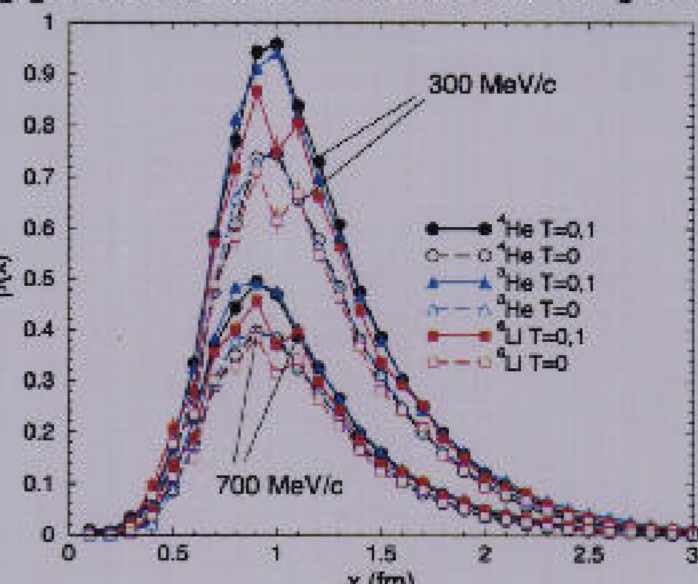
- Strength comes from $\mathbf{j}_{ij}^{\dagger}\mathbf{j}_{ij}$ terms and from single-particle two-nucleon current interference
- Pion exchange current plus currents from Delta dominate
- Transverse strength comes from np pairs
- Conjecture: can scale 'excess strength' beyond 1-body from np pair dist (90 % level)

Inclusive Response: Scaling Results

NP Pair distribution functions



Assume scaling applies to each element of pair distribution fn:



- Sum rule indicates that np pairs dominate exchange current effects; larger distances more important at finite τ.

Present/Future

- Spectra / Static Properties:
  - ◇ Larger Nuclei (GFMC up to 12; AFDMC beyond)
  - ◇ Neutron & Nuclear Matter: EOS, Long-Distance Properties
  - ◇ β decay of p-shell nuclei
- Low Energy Scattering:
  - ◇ Hadronic Reactions as tests of interaction
  - ◇ Electroweak Capture Reactions (BBN, solar, ...)
  - ◇ PV processes
  - ◇ Resonance properties in p-shell states
- Response and Beyond
  - ◇ Mass Dependence of EM response in nuclei
  - ◇ Inclusive Neutrino scattering
  - ◇ Polarizabilities, ...