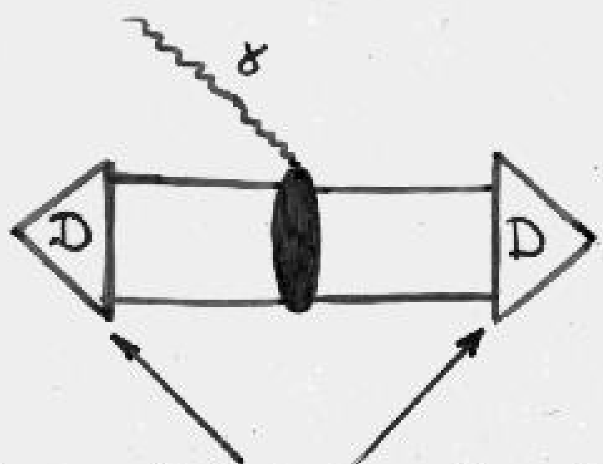
Evgeny Epelbaum, Halifax, 24.08.01

Chiral Theory Applied to Nucleon-Nucleon Systems

- Motivation
- Chiral effective theory for few nucleons: brief introduction
- Results for np scattering
- Isospin violating effects
- 4N contact interactions:
  - interpretation in terms of heavy meson exchanges
  - naturalness
  - Wigner symmetry
- Outlook



it is crutial to understand the NN system!

- ChPT ≠ solving QCD BUT: χ-symmetry QCD

- χ-symmetry is a VERY ACCURATE symmetry of QCD : $\delta \sim \frac{M_\pi^2}{\Lambda_\chi^2} \sim 2\%$
(for two flavours)

- Long-range part of the NN force
( + ...) underlies
STRONG CONSTRAINTS from χ-symm.

- Chiral symmetry + power counting = systematic expansion

— systematic theory: know how to improve

— consistent calculation of many-nucleon forces

— external sources: consistent current operators

— isospin violating effects

ChPT and nuclear forces

QCD: $$\mathcal{L}_{QCD} = \underbrace{\bar{q} i \not{D} q - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\alpha,\mu\nu}}_{\substack{SU(N_f)_L \times SU(N_f)_R\text{-} \\ \text{invariant}}} - \underbrace{\bar{q} \mathcal{M} q}_{m_u, m_d \ll \Lambda_{QCD}}$$

$SU(N_f)_A$ – spontaneously broken

$\Rightarrow N_f^2 - 1$ Goldstone bosons

Effective Theory:

Degrees of freedom: Goldstone bosons $(\pi^{\pm,0})$

+ matter fields $(N, \Delta, \dots)$

$$\mathcal{L}_{\pi\pi} = \frac{1}{2} \partial_\mu \pi \cdot \partial^\mu \pi - \frac{M_\pi^2}{2} \pi^2 + \frac{1}{2 f_\pi^2} (\pi \cdot \partial_\mu \pi)(\pi \cdot \partial^\mu \pi) + \dots$$

$$\begin{aligned} \mathcal{L}_{\pi N} = {} & N^\dagger \left(i \partial_0 + \frac{\vec{\nabla}^2}{2m} \right) N \\ & - N^\dagger \Big(\frac{1}{4 f_\pi^2} \tau \cdot (\pi \times \dot{\pi}) + \frac{g_A}{2 f_\pi} (\vec{\sigma} \tau \cdot \vec{\nabla} \pi) - \frac{2 c_1}{f_\pi^2} M_\pi^2 \pi^2 \\ & + \frac{c_3}{f_\pi^2} (\partial_\mu \pi \cdot \partial^\mu \pi) - \frac{c_4}{2 f_\pi^2} \vec{\sigma} \tau \cdot (\vec{\nabla} \pi \times \vec{\nabla} \pi) \Big) N + \dots \end{aligned}$$

$$\mathcal{L}_{NN} = - \frac{C_S}{2} (N^\dagger N)(N^\dagger N) - \frac{C_T}{2} (N^\dagger \vec{\sigma} N) \cdot (N^\dagger \vec{\sigma} N) + \dots$$

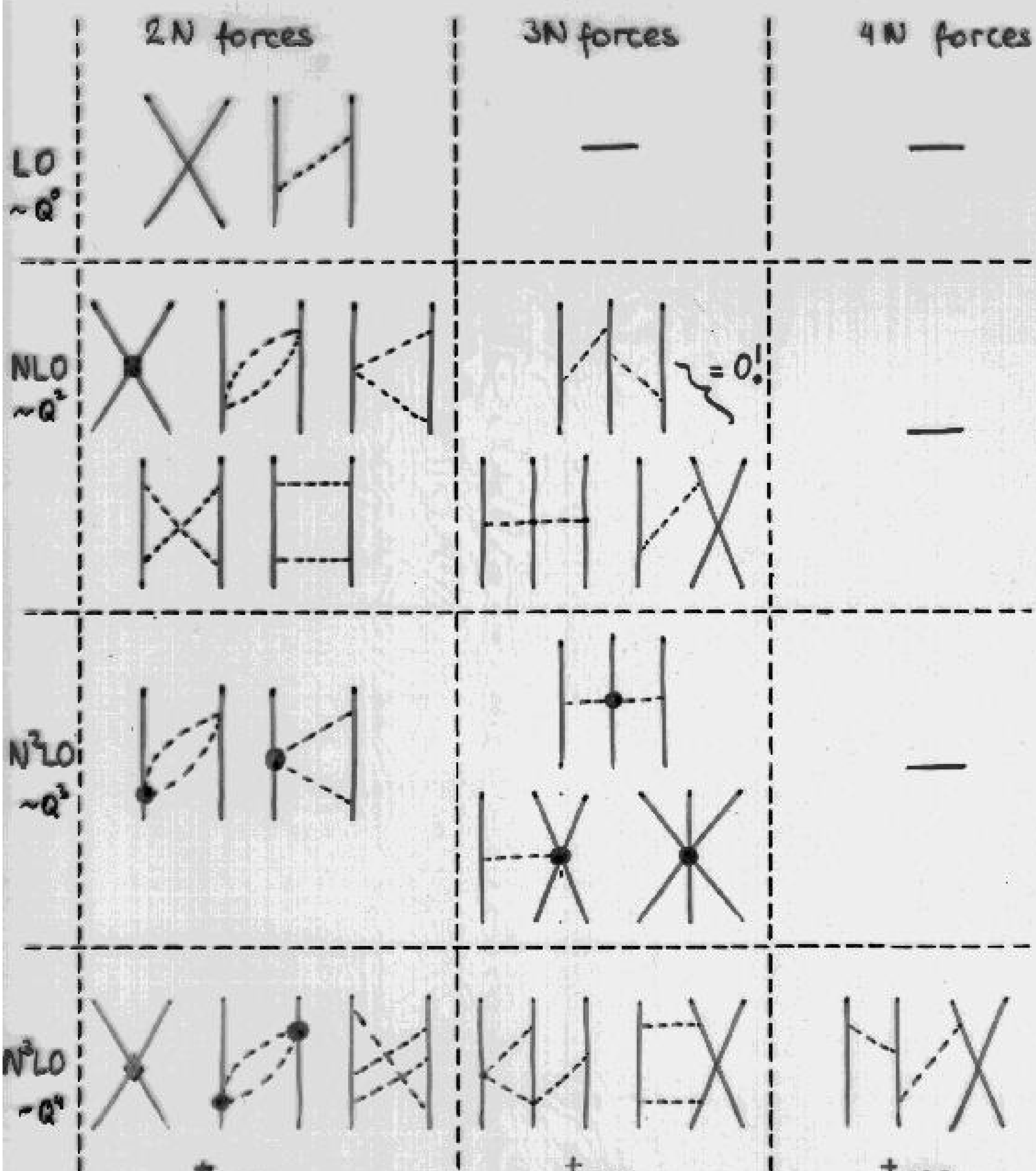
ext. nucl. loops sep. conn. pieces

Power counting: $\nu = -2 + 2E_n + 2(L - C) + \sum_i v_i \Delta_i$;

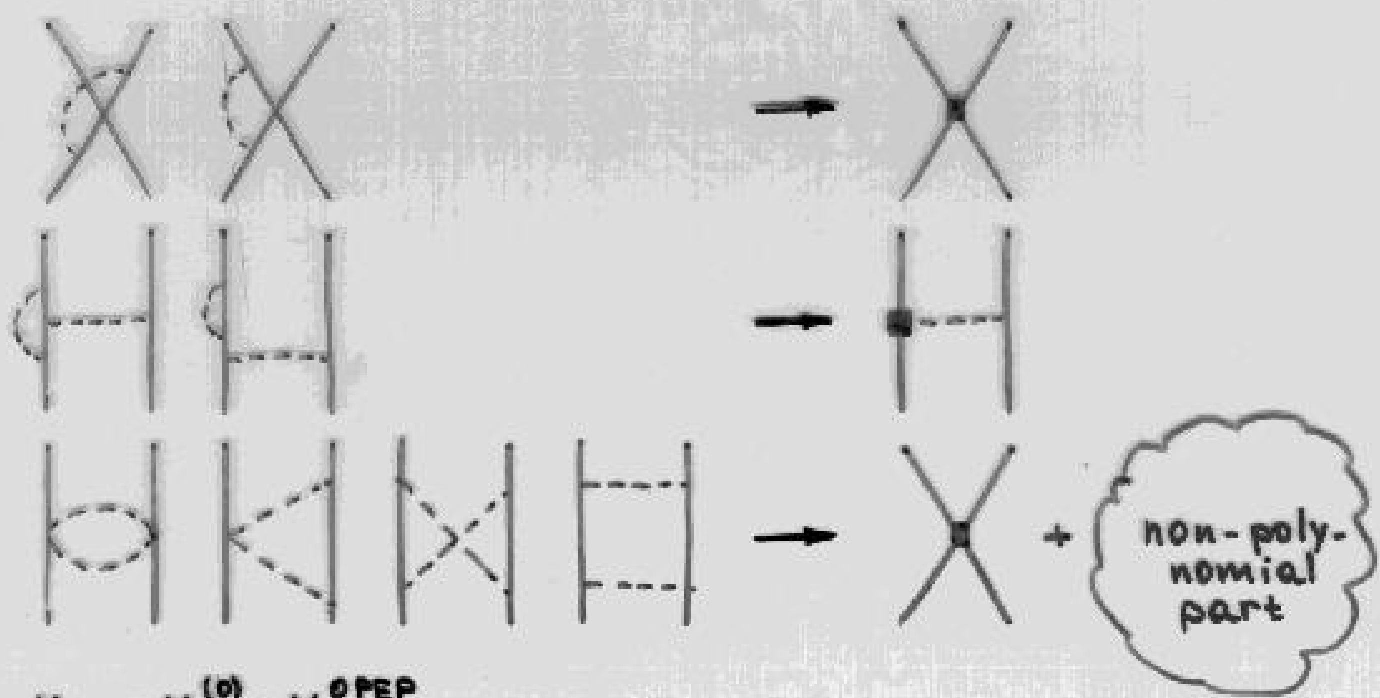
any diagram $\sim \left(\frac{Q}{\Lambda_\chi}\right)^\nu$

$\Delta_i = d_i + \frac{1}{2} n_i - 2$

derivatives or M_π nucl. fields



Renormalization



$$V_{LO} = V^{(0)} + V^{OPEP}$$

$$V^{(0)} = C_S + C_T\, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$V^{OPEP} = \left(-\frac{g_A}{2f_\pi}\right)^2 \tau_1 \cdot \tau_2 \; \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{q^2 + M_\pi^2},$$

$$\vec{q} = \vec{p}\,' - \vec{p}$$

$$V_{NLO} = V^{(0)} + V^{(2)} + V^{OPEP} + V^{TPEP}_{NLO}$$

$$V^{(2)} = C_1 \vec{q}^{\,2} + C_2 \vec{k}^2 + (C_3 \vec{q}^{\,2} + C_4 \vec{k}^2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + i C_5 \tfrac{1}{2} (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{q} \times \vec{k})$$
$$+ C_6 (\vec{q} \cdot \vec{\sigma}_1)(\vec{q} \cdot \vec{\sigma}_2) + C_7 (\vec{k} \cdot \vec{\sigma}_1)(\vec{k} \cdot \vec{\sigma}_2)$$

$$\vec{k} = \tfrac{1}{2} (\vec{p}\,' + \vec{p})$$

$$V^{TPEP}_{NLO} = -\frac{\tau_1 \cdot \tau_2}{384 \pi^2 f_\pi^4} L(q) \Big\{ 4 M_\pi^2 (5 g_A^4 - 4 g_A^2 - 1)$$
$$+ \vec{q}^{\,2} (23 g_A^4 - 10 g_A^2 - 1) + \frac{48 g_A^4 M_\pi^4}{4 M_\pi^2 + q^2} \Big\}$$
$$- \frac{3 g_A^4}{64 \pi^2 f_\pi^4} L(q) \Big\{ (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) - q^2 (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \Big\};$$

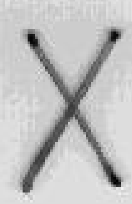
$$L(q) = \frac{1}{q} \sqrt{4 M_\pi^2 + q^2}\; \ln \frac{\sqrt{4 M_\pi^2 + q^2} + q}{2 M_\pi}; \quad q = |\vec{q}|$$

$$V_{NNLO} = V^{(0)} + V^{(2)} + V^{OPEP} + V^{TPEP}_{NLO} + V^{TPEP}_{NNLO}$$

Low energy constants in the NN potential

LO:

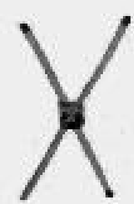
$g_A = 1.26$
$f_\pi = 92.4$ MeV
($M_\pi \cong 138$ MeV)



$\left.\begin{matrix} C_S \\ C_T \end{matrix}\right\}$ are fixed by fit to S-wave phase shifts

NLO:

no new parameters



$C_1, \dots, C_7$ are fixed by fit to S-, P-waves phase shifts and ε_1
(C_S, C_T are refitted)

N²LO:

c_1, c_3, c_4

$C_S, C_T, C_1, \dots C_7$ are refitted

What are the values of c_1, c_3, c_4?

- πN scattering 
  - Q^2: $c_1 = -0.64$ $c_3 = -3.80$ $c_4 = 2.25$
  - Q^3: $c_1 = -0.81 \dots -1.53$ $c_3 = -4.9 \dots -6.2$ $c_4 = 3.3 \dots 4.1$
  - Q^4:

	Karlsruhe	Matsinos	VPI
$\tilde{c}_1$	-2.54	-0.27	-3.31
$\tilde{c}_3$	-8.86	-1.44	-10.37
$\tilde{c}_4$	2.80	3.53	2.86

from N. Fettes, U.-G. Meißner, Nucl. Phys. A 676 (2000), 311

- NN scattering, partial-wave analysis (M.C.M. Rentmeester et al., PRL 82 (1999) 4992)

  - $c_1 = -0.77$ — fixed
  - $c_3 = -5.08$; $c_4 = 4.70$ are obtained from the fit to NN data

- Make use of very successfull B.E. models of the NN interaction

- other mesons are less importent
- no Δ-contributions

⇒ smaller values of c_i's :

$c_1 \sim -0.8$ GeV^{-1}
$c_3 \sim -1.25$ GeV^{-1}
$c_4 \sim 1.70$ GeV^{-1}

NNLO

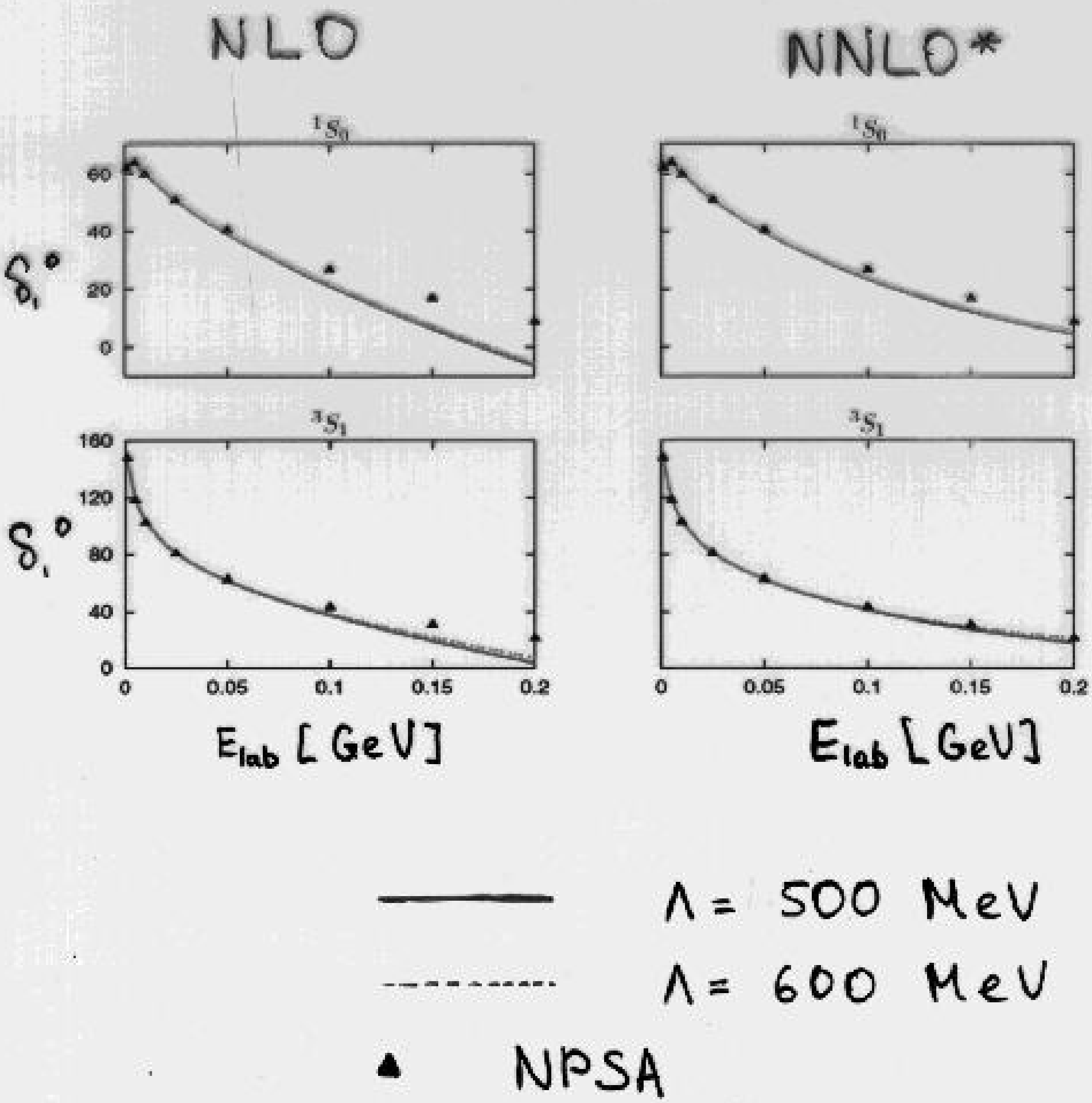
$c_1 = -0.81$
$c_3 = -4.70$
$c_4 = 3.40$
P. Büttiker et al., NPA 668 (2000) 97

- clear improvement compared to NLO
- deeply bound states in low partial waves
- good description of high partial waves
- large 3N forces

NNLO*

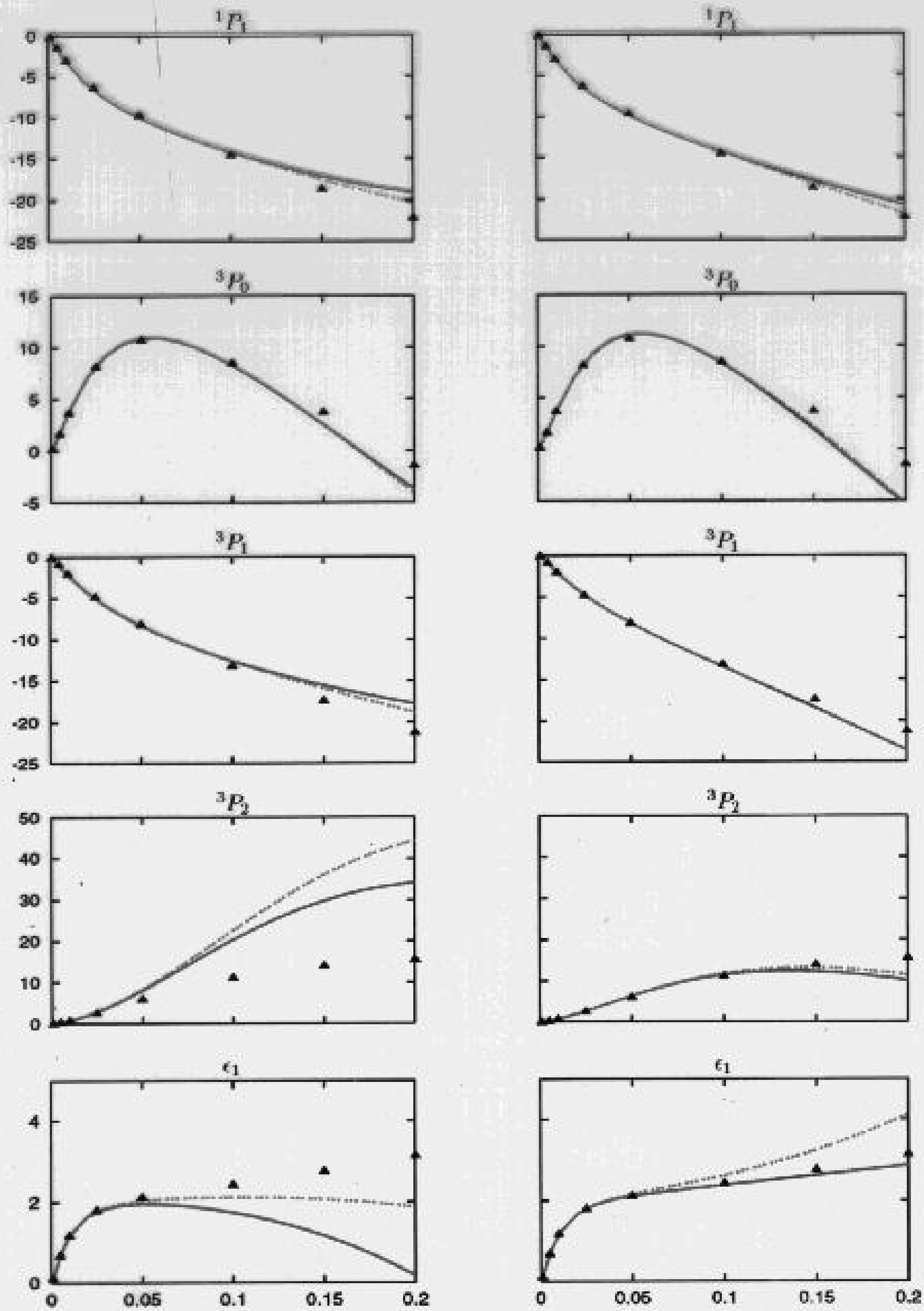
$c_1 = -0.81$
$c_3 = -1.15$ } fine tuned
$c_4 = 1.20$

- clear improvement compared to NLO
- no deeply bound states
- not so good description of high partial waves at high energies
- small 3N forces

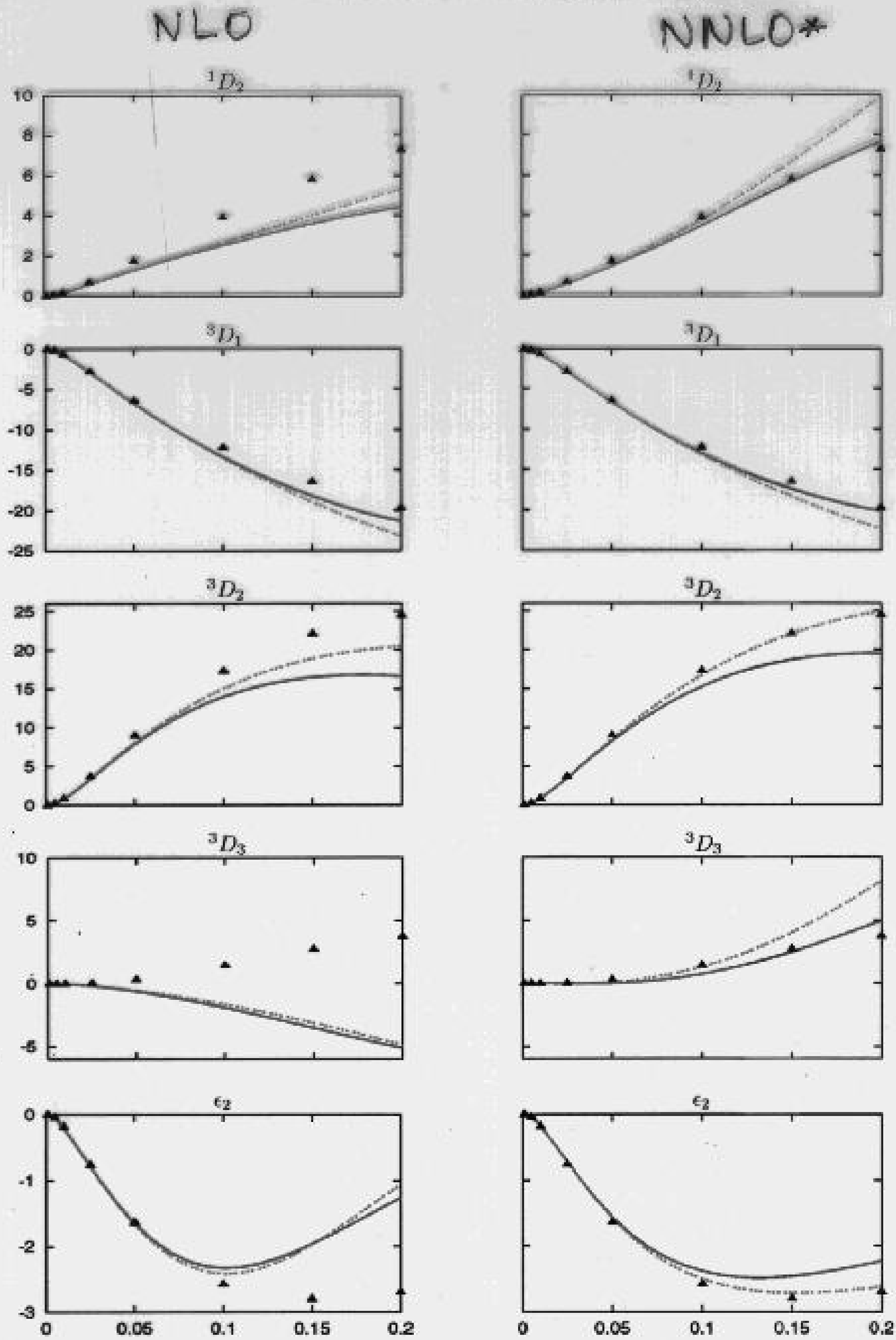


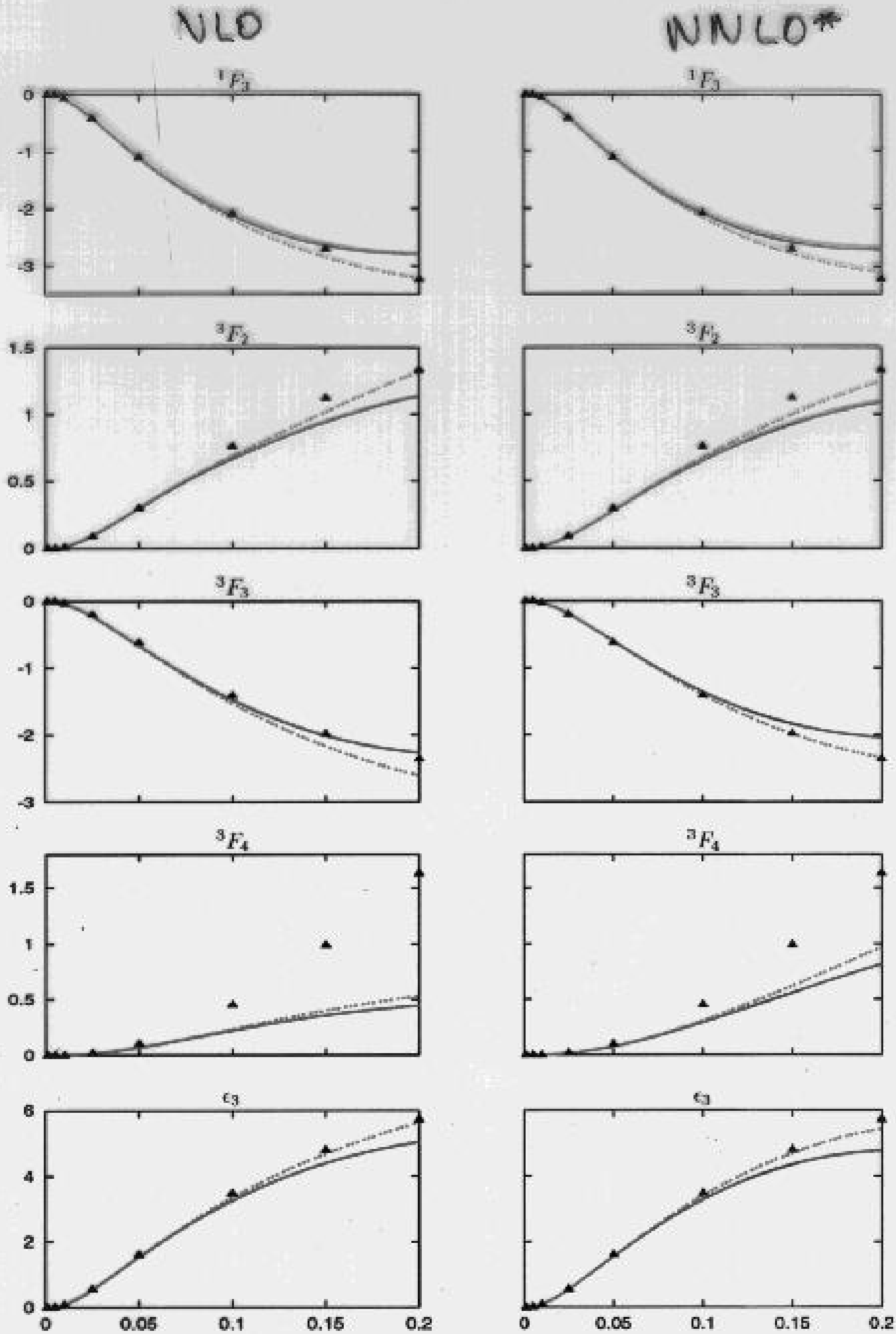
NLO
NNLO*
$^1S_0$
$^3S_1$
$\delta_1°$
E_{lab} [GeV]
0
0.05
0.1
0.15
0.2
60
40
20
160
120
80
Λ = 500 MeV
Λ = 600 MeV
NPSA

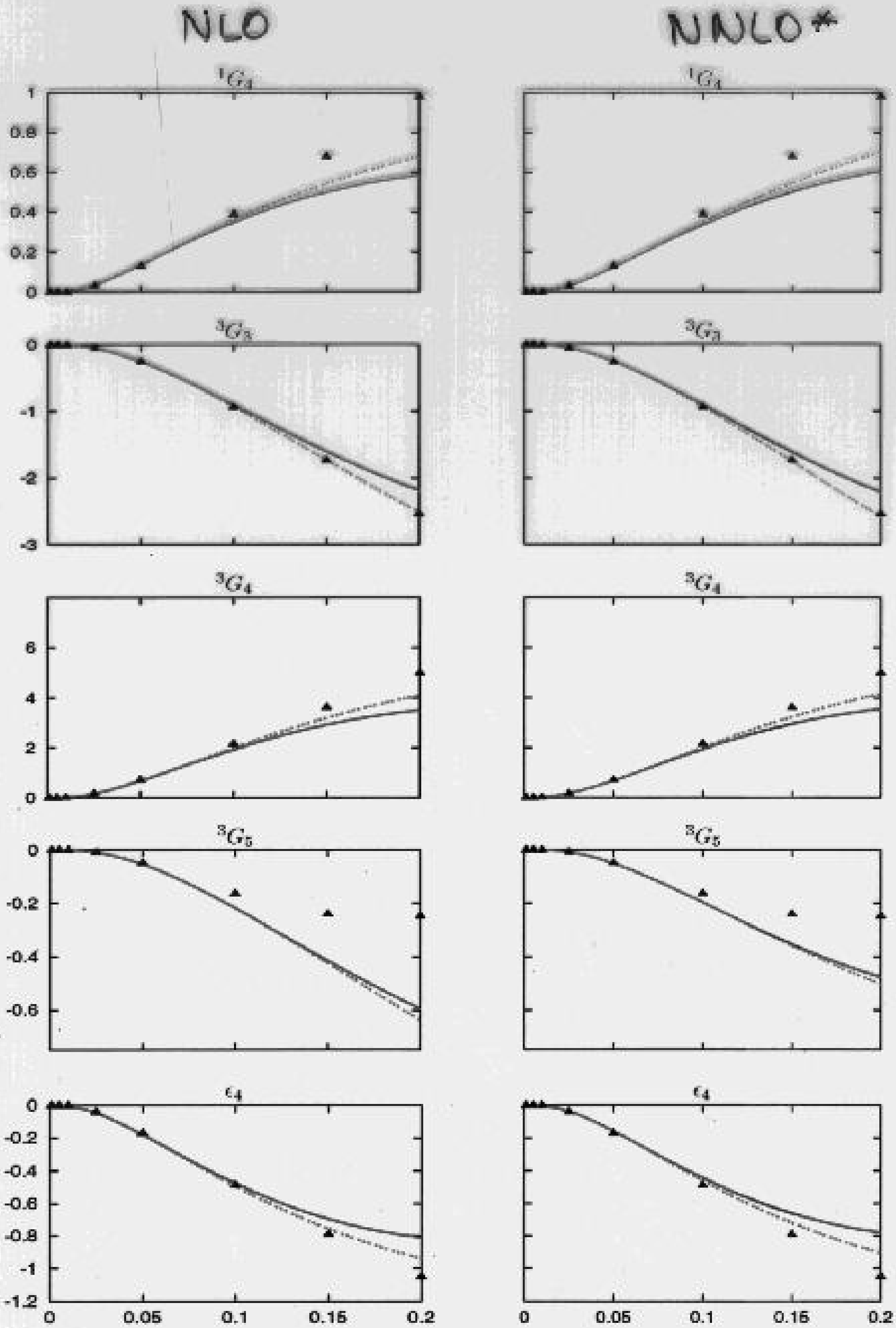
NLO NNLO*



20.09.01 epelbaum




NLO
NNLO*
$^1F_3$
$^3F_2$
$^3F_3$
$^3F_4$
ϵ_3
0
0.05
0.1
0.15
0.2


NLO
NNLO*
$^1G_4$
$^3G_3$
$^3G_4$
$^3G_5$
ϵ_4

Isospin violating effects

(M. Walzl, U.-G. Meißner, E.E., Nucl. Phys. A (2001) in press)

NN forces are charge dependent:

$$\delta_\ell^{nn} \neq \delta_\ell^{np} \neq \delta_\ell^{pp}$$

Sources of isospin violation:

- strong isospin breaking effects

$$\mathcal{H}^{QCD}_{mass} = -\frac{1}{2} \bar{q} \, (m_u + m_d) \, (1 + \varepsilon \tau_3) \, q \, ; \qquad \varepsilon \equiv \frac{m_u - m_d}{m_u + m_d} \sim \frac{1}{3}$$

- electromagnetic effects

$\sim Q \equiv \frac{e}{2} (1 + \tau_3)$ – nucleon charge matrix

Classification scheme:

LO $\left\{ \right.$
- Coulomb force
- Pion mass difference in OPE ($M_{\pi^\pm} \neq M_{\pi^0}$) ← CIB

NLO $\left\{ \right.$
- Pion mass difference in TPE ← CIB
  (J. Friar, U. van Kolck, PRC 60 (1999) 034006)
- $\pi\gamma$ – exchange ← CIB
  (U. van Kolck et al., PRL 80 (1998) 4386)
- Contact term $(N^\dagger \tau_3 N)(N^\dagger \tau_3 N)$ ← CIB
- Contact term $(N^\dagger \tau_3 N)(N^\dagger N)$ ← CSB

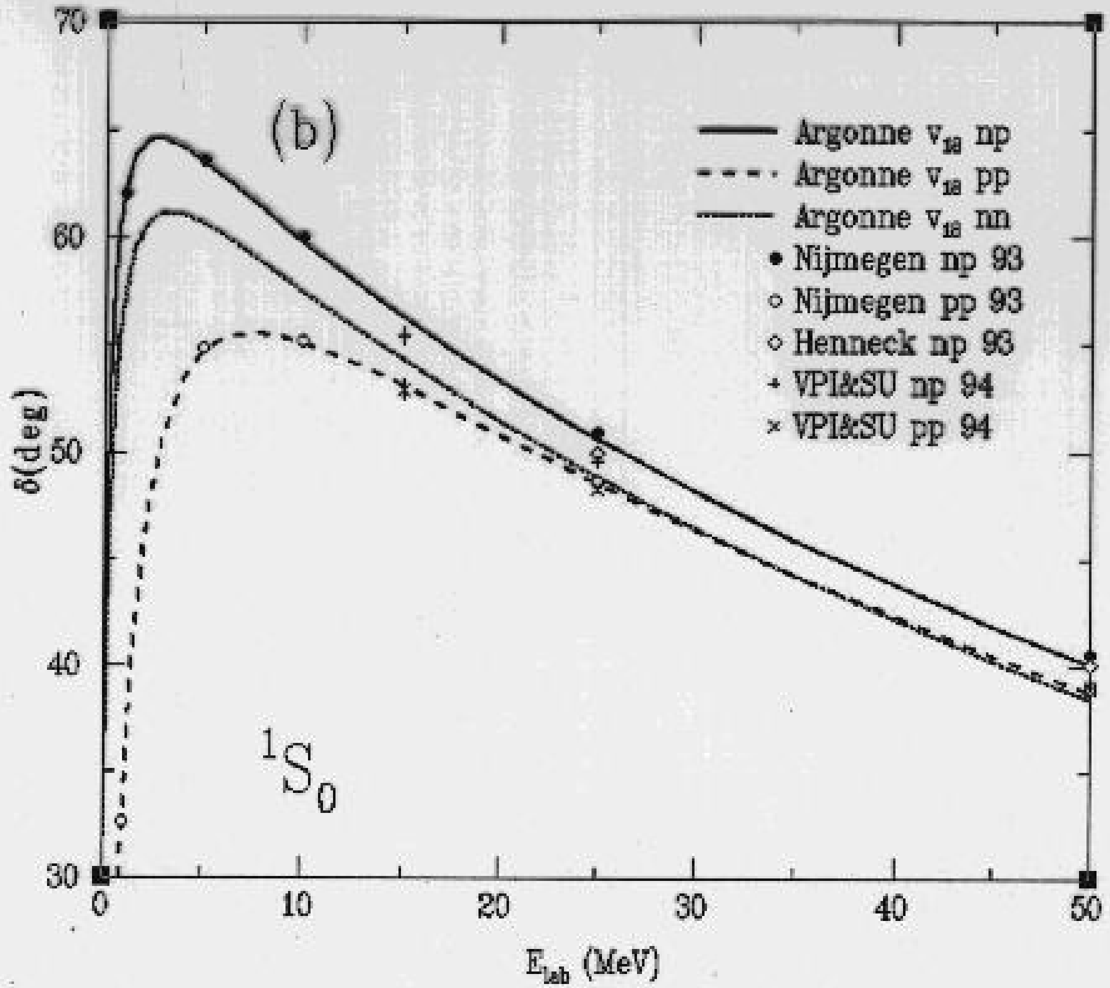
$C_{CIB} (N^\dagger \tau_3 N)(N^\dagger \tau_3 N)$ $\qquad$ $C_{CSB} (N^\dagger \tau_3 N)(N^\dagger N)$

are fixed using exp. values of a^C_{pp}, a_{nn}

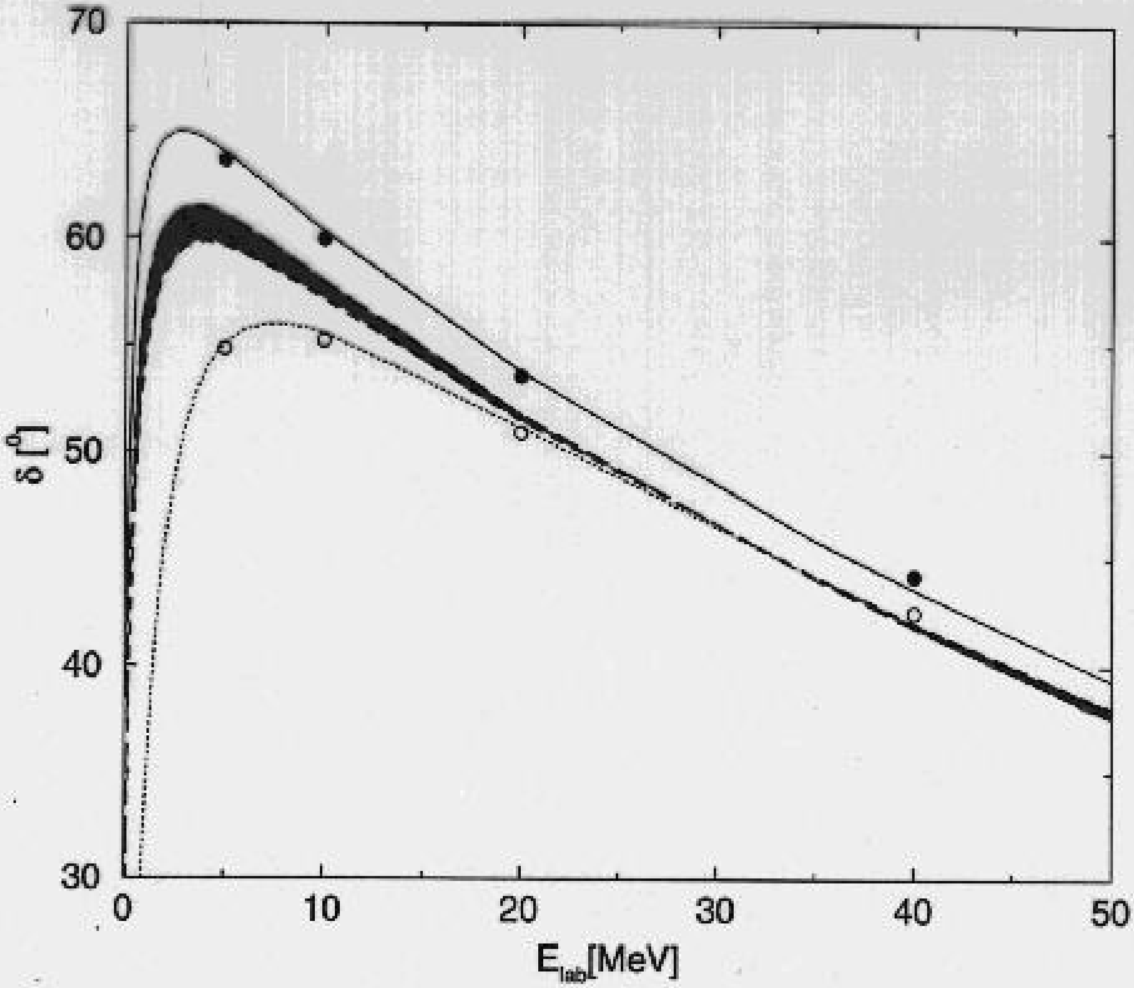
Predictions

- energy dependence of the $^1S_0$ phase shift
- CIB in higher waves (parameter free)

$^1S_0$ partial wave for nn, np, pp



(from: http://www.phy.anl.gov/theory/research/av18)



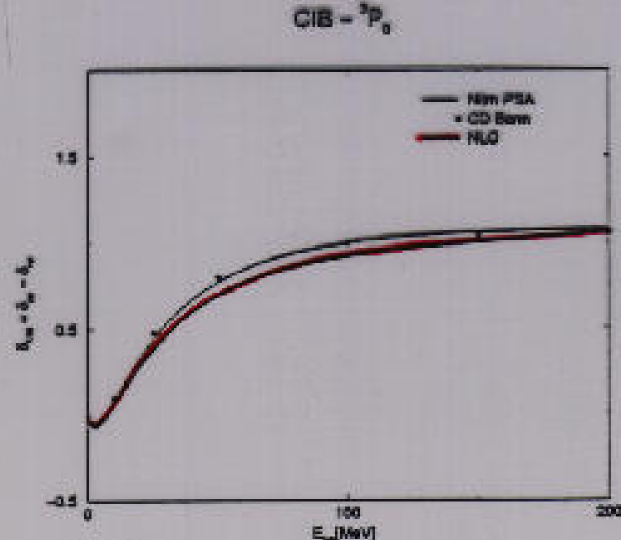
np

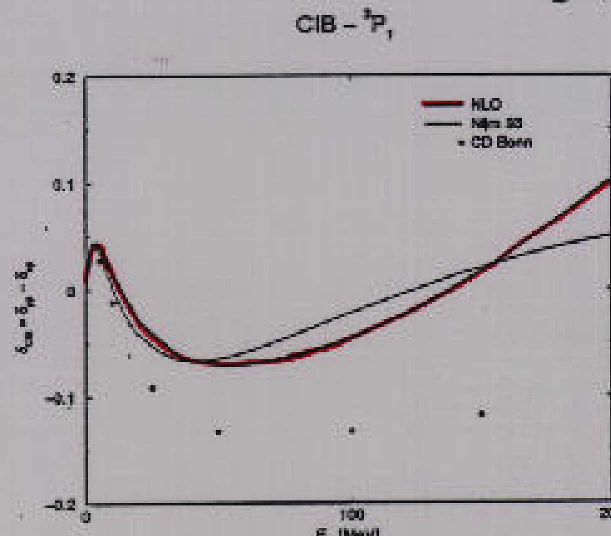
pp

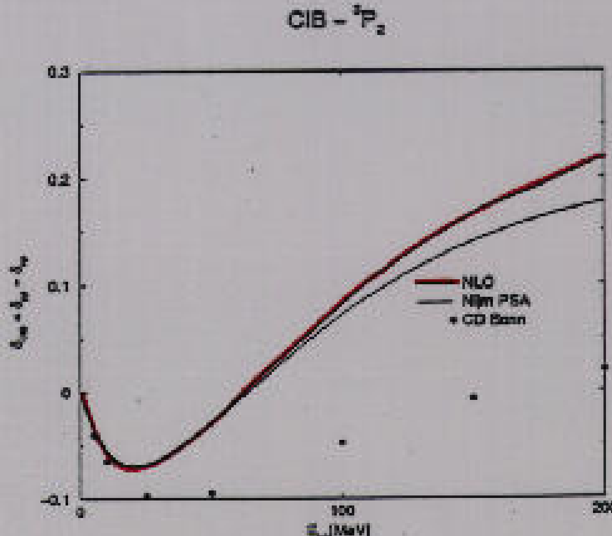
nn, Slaus (89)

nn, Bonn (00)

Charge independence breaking in the ^3P_J - waves







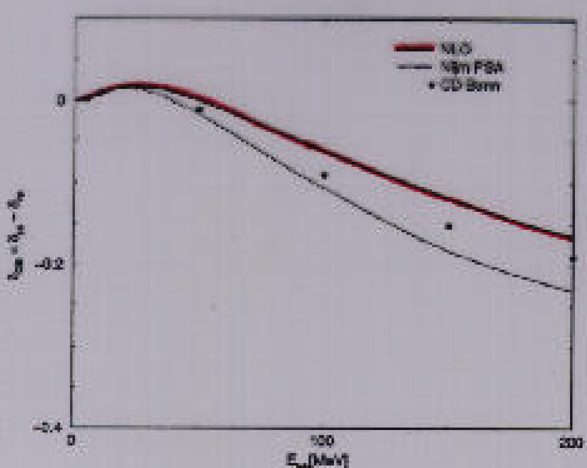
NLO

Nijmegen PSA

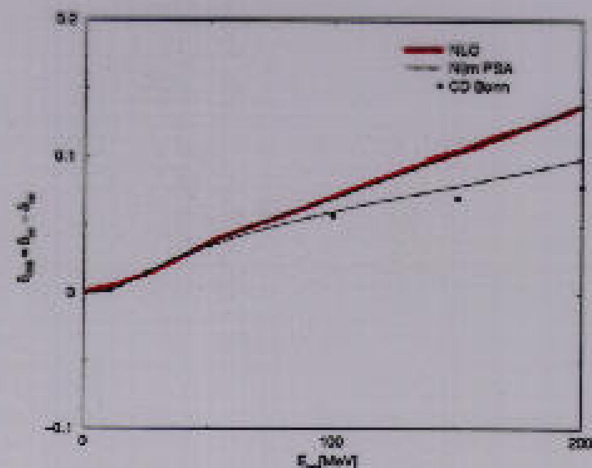
CD Bonn

Charge independence breaking in higher partial waves

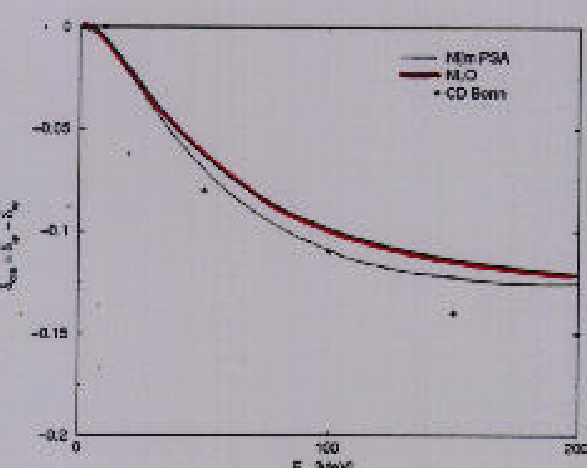
CIB – $^1D_2$



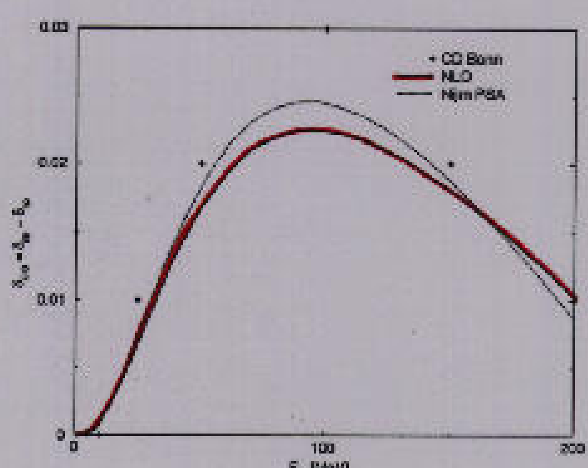
CIB – $^3F_2$



CIB – $^3F_3$



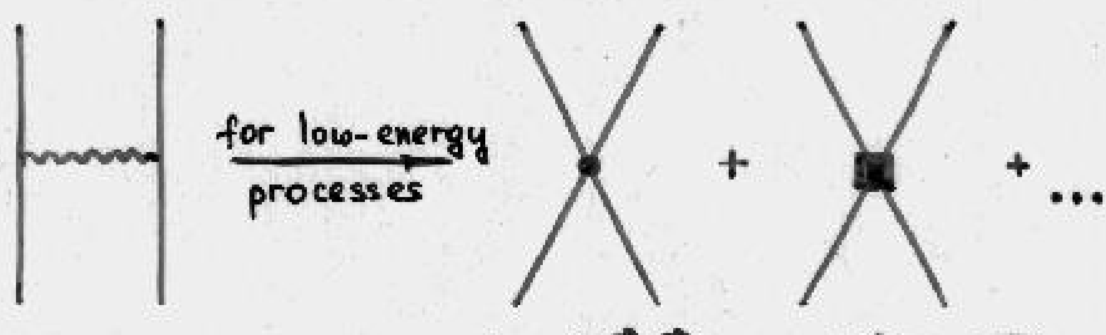
CIB – $^1G_4$



Physical interpretation of the 4N operators (resonance saturation)

E.E., Ulf-G. Meißner, W. Glöckle, Ch. Elster,
nucl-th/0106007

Idea: use boson exchange models of the NN interaction (Bonn B, Nijm. 93) to estimate the size of the 4N contact operators



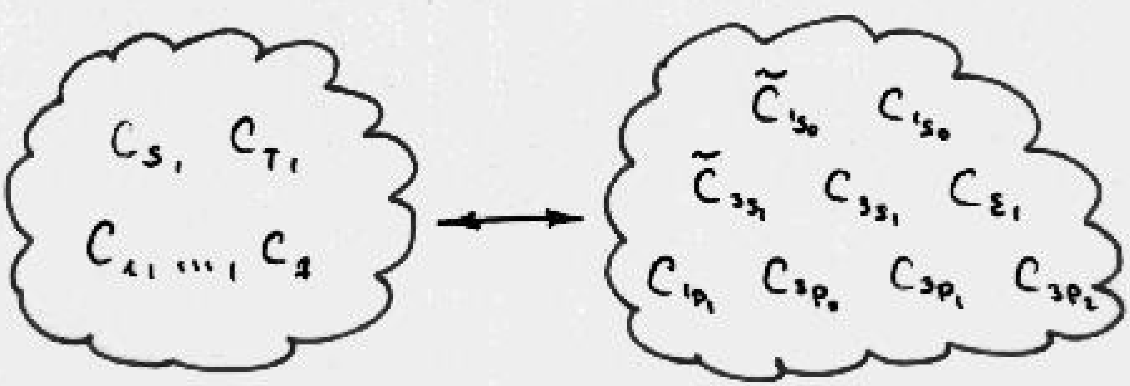
$\rho, \omega, \ldots$ $\qquad$ $C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$ $\qquad$ $C_1 \vec{q}^{\,2} + C_2 \vec{k}^2 + \ldots$

Contact operators

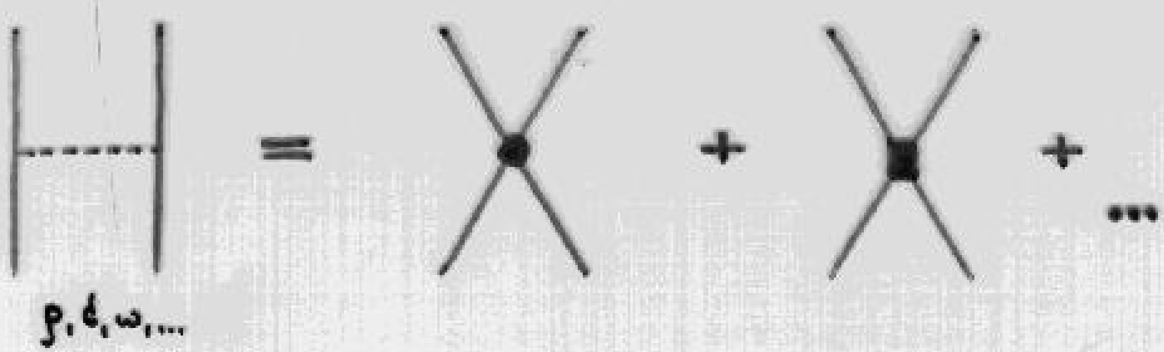
$$
\begin{aligned}
\mathcal{L}_{NN} &= \mathcal{L}_{NN}^{(2)} + \mathcal{L}_{NN}^{(4)} + \ldots, \\
\mathcal{L}_{NN}^{(2)} &= -\frac{1}{2} C_S \,(N^\dagger N)\,(N^\dagger N) - \frac{1}{2} C_T \,(N^\dagger \sigma_i N)(N^\dagger \sigma_i N)\,, \\
\mathcal{L}_{NN}^{(4)} &= -\frac{1}{2} C_1 \left[(N^\dagger \partial_i N)^2 + ((\partial_i N^\dagger) N)^2 \right] - \left(C_1 - \frac{1}{4} C_2 \right) (N^\dagger \partial_i N)((\partial_i N^\dagger) N) \\
&\quad + \frac{1}{8} C_2 (N^\dagger N) \left[N^\dagger \partial_i^2 N + \partial_i^2 N^\dagger N \right] \\
&\quad - \frac{i}{8} C_5 \epsilon_{ijk} \Big\{ \left[(N^\dagger \partial_i N)((\partial_j N^\dagger) \sigma_k N) + ((\partial_i N^\dagger) N)(N^\dagger \sigma_j \partial_k N) \right] \\
&\quad - (N^\dagger N)((\partial_i N^\dagger) \sigma_j \partial_k N) + (N^\dagger \sigma_i N)((\partial_j N^\dagger) \partial_k N) \Big\} \\
&\quad + \frac{1}{4} \left(\left(C_6 + \frac{1}{4} C_7 \right) (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{kj}) + \left(2C_3 + \frac{1}{2} C_4 \right) \delta_{ij}\delta_{kl} \right) \\
&\qquad \times \left[((\partial_i \partial_j N^\dagger) \sigma_k N) + (N^\dagger \sigma_k \partial_i \partial_j N) \right] (N^\dagger \sigma_l N) \\
&\quad - \frac{1}{2} \left(C_6 \,(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{kj}) + C_4 \delta_{ij}\delta_{kl} \right) (N^\dagger \sigma_k \partial_i N)((\partial_j N^\dagger) \sigma_l N) \\
&\quad - \frac{1}{8} \left(\frac{1}{2} C_7 \,(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{kj}) - (4C_3 - 3C_4) \delta_{ij}\delta_{kl} \right) \\
&\qquad \times \left[(\partial_i N^\dagger \sigma_k \partial_j N) + (\partial_j N^\dagger \sigma_k \partial_i N) \right] (N^\dagger \sigma_l N)
\end{aligned}
$$

$$
\begin{aligned}
V_{\rm cont} &= C_S + C_T (\vec\sigma_1 \cdot \vec\sigma_2) + C_1 \vec q^{\,2} + C_2 \vec k^{\,2} + (C_3 \vec q^{\,2} + C_4 \vec k^{\,2})(\vec\sigma_1 \cdot \vec\sigma_2) \\
&\quad + i C_5 \frac{\vec\sigma_1 + \vec\sigma_2}{2} \cdot (\vec q \times \vec k) + C_6 (\vec q \cdot \vec\sigma_1)(\vec q \cdot \vec\sigma_2) + C_7 (\vec k \cdot \vec\sigma_1)(\vec k \cdot \vec\sigma_2)\,,
\end{aligned}
$$

where $\vec q = \vec p\,' - \vec p$ and $\vec k = (\vec p + \vec p\,')/2$



Boson exchange models and "realistic" forces



LEC	Bonn-B	CD-Bonn*	Nijm-93	Nijm-I*	Nijm-II*	AV-18*
$\tilde{C}_{1S0}$	−0.117	−0.140	−0.061	−0.137	−0.091	−0.037
C_{1S0}	1.276	1.388	1.426	1.391	1.357	1.409
$\tilde{C}_{3S1}$	−0.101	−0.103	−0.014	−0.058	0.029	0.026
C_{3S1}	0.660	0.869	0.940	0.762	0.795	0.867
$C_{\epsilon 1}$	−0.410	−0.315	−0.343	−0.221	−0.241	−0.226
C_{1P1}	0.454	0.228	0.119	0.328	0.401	0.290
C_{3P0}	0.921	0.956	0.802	0.802	0.949	0.723
C_{3P1}	−0.075	−0.051	−0.197	−0.059	−0.075	0.067
C_{3P2}	−0.396	−0.451	−0.513	−0.453	−0.451	−0.467

keep OPE as it is and expand all other short-range contributions in powers of momenta $\Rightarrow$ $\tilde{C}_{^1S_0}, C_{^1S_0}, \ldots, C_{^3P_2}$

OBE models

- Bonn B

LEC	η	σ	δ	ω	ρ	sum
$\tilde{C}_{1S0}$	0.000	−0.392	−0.023	0.287	0.011	−0.117
C_{1S0}	0.033	1.513	0.036	−0.560	0.254	1.276
$\tilde{C}_{3S1}$	0.000	−0.424	0.070	0.287	−0.034	−0.101
C_{3S1}	−0.011	1.030	−0.108	−0.777	0.526	0.660
$C_{\epsilon 1}$	−0.032	0.000	0.000	0.077	−0.455	−0.410
C_{1P1}	−0.022	−0.607	0.059	0.536	0.488	0.454
C_{3P0}	−0.067	−0.786	−0.011	1.187	0.597	0.921
C_{3P1}	0.045	−0.860	−0.015	0.753	0.003	−0.075
C_{3P2}	0.000	−1.008	−0.024	0.536	0.101	−0.396

pseudo-scalar (η) — scalar (σ, δ) — vector (ω, ρ)

- Nijm 93

LEC	η	η'	ρ	ω	ϕ	a_0	ϵ	f_0	a_2	Pom.
$\tilde{C}_{1S0}$	0.000	0.000	0.020	0.237	0.001	−0.031	−0.578	−0.201	0.001	0.490
C_{1S0}	0.041	0.013	0.191	−0.445	−0.002	0.134	3.461	0.867	−0.005	−2.83
$\tilde{C}_{3S1}$	0.000	0.000	−0.055	0.237	0.001	0.094	−0.578	−0.201	−0.003	0.490
C_{3S1}	−0.014	−0.004	0.550	−0.700	−0.003	−0.403	3.461	0.867	0.015	−2.83
$C_{\epsilon 1}$	−0.038	−0.012	−0.383	0.090	0.000	0.000	0.000	0.000	0.000	0.000
C_{1P1}	−0.027	−0.009	0.431	0.423	0.002	0.250	−2.194	−0.539	−0.010	1.791
C_{3P0}	−0.082	−0.026	0.645	1.167	0.006	−0.072	−1.985	−0.466	0.003	1.613
C_{3P1}	0.054	0.017	0.032	0.660	0.003	−0.078	−2.087	−0.501	0.003	1.700
C_{3P2}	0.000	0.000	0.114	0.457	0.002	−0.090	−2.308	−0.579	0.003	1.887

pseudoscalar (η, η') — vector (ρ, ω, ϕ) — scalar mesons (a_0, ϵ, f_0) — Regge traject. scalar (a_2, Pom.)

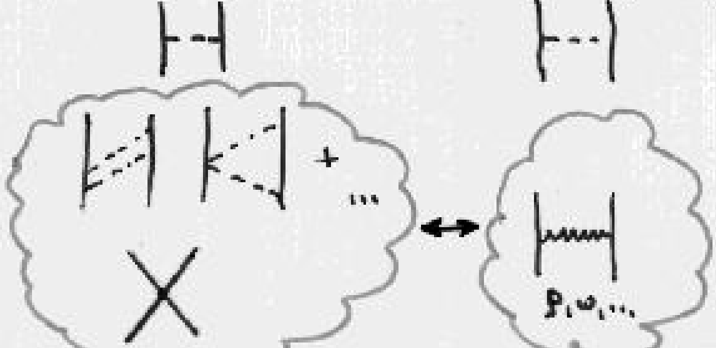
- chiral potential

LEC	TPE(NLO)	TPE(NNLO)	C_i (NLO)	C_i (NNLO)
$\tilde{C}_{1S0}$	-0.004	0.003	$-0.156 \ldots -0.110$	$-0.160 \ldots -0.158$
C_{1S0}	-0.585	-0.070	$1.048 \ldots 1.253$	$1.135 \ldots 1.134$
$\tilde{C}_{3S1}$	0.013	0.001	$-0.155 \ldots -0.023$	$-0.159 \ldots -0.134$
C_{3S1}	0.653	-0.181	$0.250 \ldots 0.840$	$0.637 \ldots 0.587$
$C_{\epsilon 1}$	-0.195	0.117	$-0.302 \ldots -0.384$	$-0.369 \ldots -0.326$
C_{1P1}	-0.069	-0.099	$0.260 \ldots 0.273$	$0.234 \ldots 0.268$
C_{3P0}	-0.436	-0.071	$0.800 \ldots 0.855$	$0.727 \ldots 0.857$
C_{3P1}	0.252	0.011	$-0.126 \ldots -0.093$	$-0.141 \ldots 0.026$
C_{3P2}	-0.023	0.036	$-0.325 \ldots -0.259$	$-0.464 \ldots -0.445$

- OBE models & high-precision potentials

LEC	Bonn-B	CD-Bonn*	Nijm-93	Nijm-I*	Nijm-II*	AV-18*
$\tilde{C}_{1S0}$	-0.117	-0.140	-0.061	-0.137	-0.091	-0.037
C_{1S0}	1.276	1.388	1.426	1.391	1.357	1.409
$\tilde{C}_{3S1}$	-0.101	-0.103	-0.014	-0.058	0.029	0.026
C_{3S1}	0.660	0.869	0.940	0.762	0.795	0.867
$C_{\epsilon 1}$	-0.410	-0.315	-0.343	-0.221	-0.241	-0.226
C_{1P1}	0.454	0.228	0.119	0.328	0.401	0.290
C_{3P0}	0.921	0.956	0.802	0.802	0.949	0.723
C_{3P1}	-0.075	-0.051	-0.197	-0.059	-0.075	0.067
C_{3P2}	-0.396	-0.451	-0.513	-0.453	-0.451	-0.467

- chiral
- OBE, high-precis. pot.



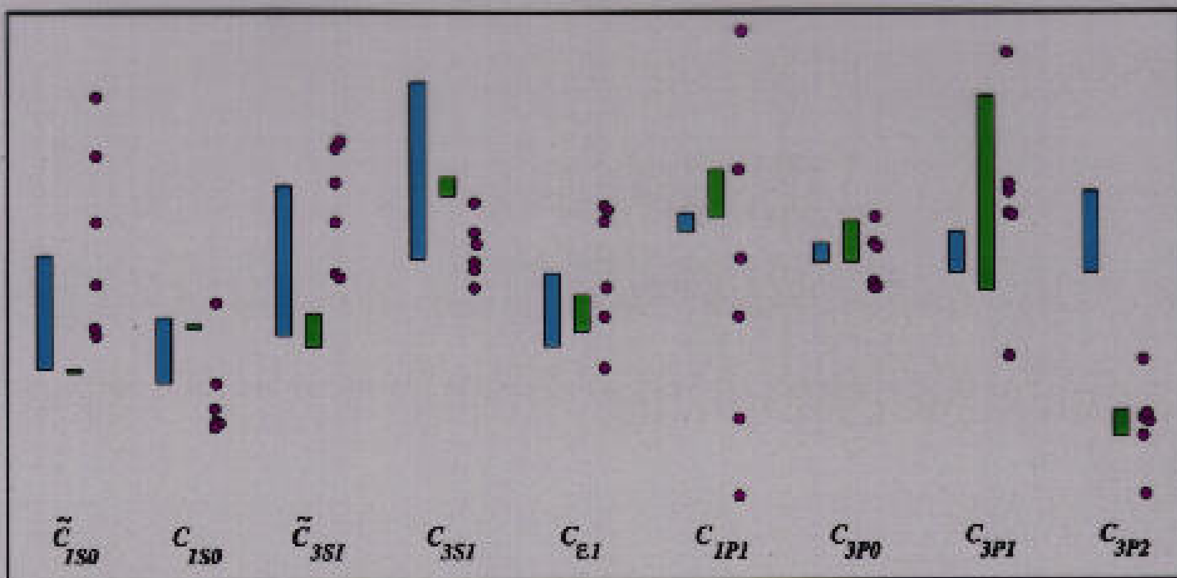


FIG. 3. Graphical representation to compare the LECs determined in EFT with the values obtained from the various models. The left bar refers to NLO, the middle bar to NNLO and the filled circles to the various potentials discussed in the text. Note that the units are arbitrary, i.e. different scale factors (including in some cases an overall minus sign) have been assigned to the various LECs to obtain a more uniform representation.

Naturalness of the contact interactions

	NLO	NNLO
$f_\pi^2 C_S$	$-1.053\ldots-0.303$	$-1.079\ldots-0.953$
$f_\pi^2 C_T$	$-0.002\ldots0.147$	$0.002\ldots0.040$
$f_\pi^2\Lambda_\chi^2 C_1$	$1.707\ldots3.162$	$3.143\ldots2.665$
$4\,f_\pi^2\Lambda_\chi^2 C_2$	$1.348\ldots3.246$	$2.029\ldots2.251$
$f_\pi^2\Lambda_\chi^2 C_3$	$-0.047\ldots-0.315$	$0.403\ldots0.281$
$4\,f_\pi^2\Lambda_\chi^2 C_4$	$-0.583\ldots-0.933$	$-0.364\ldots-0.428$
$2\,f_\pi^2\Lambda_\chi^2 C_5$	$2.418\ldots2.314$	$2.846\ldots3.410$
$f_\pi^2\Lambda_\chi^2 C_6$	$-0.385\ldots-0.651$	$-0.728\ldots-0.668$
$4\,f_\pi^2\Lambda_\chi^2 C_7$	$-1.790\ldots-2.120$	$-1.929\ldots-1.681$

$$C_S,\, C_T \sim \frac{\alpha_i}{f_\pi^2}\;; \qquad C_1, \ldots, C_7 \sim \frac{\alpha_i}{f_\pi^2\Lambda_\chi^2}$$

$$\boxed{\alpha_i \sim 1}$$

$$\mathcal{L} = \alpha_{\ell m n}\left(\frac{N^\dagger(\ldots)N}{f_\pi^2\Lambda_\chi}\right)^{\ell}\left(\frac{\pi}{f_\pi}\right)^{m}\left(\frac{\partial^\mu,\, M_\pi}{\Lambda_\chi}\right)^{n} f_\pi^2\Lambda_\chi^2$$

(J.L. Friar, Few-Body Syst. 22 (1997) 161)

Outlook

- including 3N force $\Rightarrow$ complete NNLO calculation for 3N, 4N,... systems

- meson exchange currents
- complete N^3LO calculation in the NN system